MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Rev. T. E. R. PHILLIPS, PRESIDENT, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

Charles Edward Barns, The Pacific Science Press, Morgan Hill, California, U.S.A. ;

Alfred Dillon Bell, Waihemo, Palmerston South, Otago, New Zealand ;

Hugh G. Boutell, 3723 Jocelyn Street, Washington, D.C., U.S.A. ;

Boris P. Gerasimovič, Harvard College Observatory, Cambridge, Mass., U.S.A. ;

Percy William Glover, c/o Boys' High School, New Plymouth, New Zealand ;

Sydney Goldstein, Ph.D., St. John's College, Cambridge ;

Yusuke Hagihara, Assistant Professor of Astronomy in the Imperial University of Tokyo, Astronomical Observatory, Tokyo, Japan ;

Leon Hufnagel, Ph.D., The Observatory, Lund, Sweden, and Harvard College Observatory, Cambridge, Mass., U.S.A. ;

Ronald Alexander McIntosh, 1 Melford Street, Ponsonby, Auckland, New Zealand ;

Thomas Pattinson Marshall, Lingfield, 74 Park Avenue, Hull, Yorkshire ;

Jeffrey Charles Percy Miller, B.A., Trinity College, Cambridge ;

Joseph Pearce, 72 Barclay Road, Leytonstone, London, E. 11 ;

Ralph Ernest Sadler, A.M.I.C.E., The Mansion, Ashbourne, Derbyshire ;

Paul Slavenas, Ph.D., Kaunas University, Lithuania ; and

Otto Struve, Ph.D., Assistant Professor in the University of Chicago, Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Rev. Dr. Paul A. McNally, S.J., Director of the Georgetown College Observatory, Washington, D.C., U.S.A. (proposed by H. H. Turner); and

Cyril Young, c/o Sir Howard Grubb, Parsons & Co., Heaton Works, Newcastle-on-Tyne (proposed by Sir Charles Parsons).

Sixty-nine presents were announced as having been received since the last meeting, including, amongst others :—

Catania, Reale Osservatorio : Catalogo Astrofotografico, vol. 7, parte 2 (fasc. 50); vol. 8, parte 2 (fasc. 58).

The President announced that the gold medal of the Society had been awarded to Professor Ejnar Hertzsprung, for his determination of the distance of the Lesser Magellanic Cloud and other pioneering work in stellar astronomy.

Professor Hertzsprung has been appointed George Darwin Lecturer for 1929.

On the Rotation of the Stars. By G. Shajn and O. Struve.

1. Captain W. de W. Abney (1) was, we believe, the first to express (in 1877) the idea that the axial rotation of the stars could be determined from measurements of the widths of spectral lines. This opinion met with severe criticism on the part of H. C. Vogel (2), who pointed out that the great width of the hydrogen lines in certain stars could not possibly be due to rotation, since other lines in the same spectra usually appear narrow. It is now known, however, that in many stars all lines are broad, much broader in fact than the limiting width imposed by the resolving power of the spectrograph.

Professor Frank Schlesinger (3) was the first to actually observe the rotation of stars. This he did by measuring the limb-effect in the eclipsing variables δ Libræ and λ Tauri. The same problem was later discussed by G. Forbes (4).

In 1922 J. Hellerich (5) published an important paper on the rotational effects as observed during the time of eclipse in a number of Algol-type variables.

A complete investigation of the rotational effect in β Lyræ and in

Algol was carried out by R. A. Rossiter (6) and Dean B. McLaughlin (7). More recently J. S. Plaskett (8) has shown that the effect is also present in 21 Cassiopeiæ, and McLaughlin (9) has rediscussed the eclipsing variable λ Tauri on the basis of Ann Arbor spectrograms.

All the investigations on eclipsing binaries mentioned above prove that the absorption lines are actually widened by rotation. Adams and Joy (10) have successfully predicted in a number of cases that spectroscopic binaries with very wide and diffuse lines have very short periods. Since it is probable that the rotational periods in spectroscopic binaries are equal to their orbital periods, it is clear that very short periods should be associated with wide and diffuse lines. A similar effect was also noticed by Miss Antonia C. Maury (11) in the spectroscopic binaries μ^1 Scorpii and V Puppis.

The contour of spectral lines in a rotating or pulsating star was investigated by H. Shapley and S. B. Nicholson (12) in connection with Cepheid variables.

The important theoretical discussion by J. A. Carroll (13) on the form of an absorption line in the spectrum of a rotating or expanding star unfortunately arrived only after most of our own work had been completed and prepared for publication. Both papers show that the intensities of spectral lines form a sensitive criterion for the detection of rotation. Additional sides of the problem in question are considered in this paper; we have particularly emphasized certain practical applications.

J. H. Jeans (14) has recently shown that the flow of radiation, carrying with it momentum from one part of the star to another, seriously affects the internal motions of the stars, and in the first place their axial rotations. This factor of radiative viscosity, so called because of its analogy with the ordinary viscosity of gases, leads to the interesting result that the stars do not rotate as solid bodies, but that each spherical shell has its own angular velocity roughly proportional to r^{-2}. Only the innermost portions of the stars deviate markedly from this law. The period of rotation of the outer layer is approximately nine times longer than that of the inner layers. One would, therefore, expect that the effect of axial rotation upon the spectrum would be slight. This, however, may not be true for close double stars. Immediately after fission (if double stars really originate through fission) the periods of axial rotation and of orbital revolution must be identical.

There is also observational evidence in support of this equality. Thus in the β Lyræ stars the light-variation is continuous, indicating that the components are nearly in contact. A difference between the two periods would result in marked changes in the light curve from one revolution to the next, this being caused by the elliptical shape of the components. No such changes have been observed.

One of the stars in which Adams and Joy (15) suspect rotational widening of the lines is the well-known variable W Ursæ Majoris. Being a dwarf of spectral type F8p, the radii of its components cannot be very large. Adams and Joy found that the greatest semi-axis of each component is 540,000 km. or 0·78 in terms of the Sun's radius. The total

mass of this system is very nearly that of the Sun, and the period is one of the shortest yet discovered, $0^d{\cdot}334$. The lines are described as "so widened and weakened that measures for velocity and estimates of line-intensity can be made only with the greatest difficulty." It is suggested that "the unusual character of the spectral lines is due mainly to the rotational effect in each star, which may cause a difference of velocity in the line of sight of as much as 240 km. between the two limbs of the star." In this connection an interesting attempt was made by J. Schilt (16) to consider W Ursæ Majoris as a rotating star just preceding fission.

It can be shown that if the rotational velocity of W Ursæ Majoris produces appreciable widening of the lines and decrease in their depth (or central intensity), a similar effect should exist in binaries of early spectral type and of somewhat longer period (17).

Let ω be the angular velocity of the star, let the radius of the star, in km.'s, be designated as r (we neglect here any possible deformations due to tidal effects), and let the inclination of the orbit be i. The actual rotational velocity will then be

$$V_{rot.} = r\frac{d\theta}{dt} = r\omega = \frac{r}{P} \cdot \text{const.},$$

and the observed component is

$$V_0 = \frac{r}{P} \cdot \sin i \cdot \text{const.} \quad . \quad . \quad . \quad (1)$$

We now make the assumption that in close spectroscopic binaries the periods of rotation and of revolution are actually identical, so that the value of P in our formula is given by the observations.

Consider the specific case of a spectroscopic binary of type Bo having an absolute magnitude of $-3{\cdot}0$ for the brighter component and an orbital period of $1^d{\cdot}5$. The approximate diameter of the brighter component is, according to Russell's tables (18), about eight times that of the Sun, or ten times that of the diameter of each component in W Ursæ Majoris. Consequently the term $\frac{r}{P}$ is about 2·2 times larger in our hypothetical star than in W Ursæ Majoris. Provided that sin i is in both cases the same, the spectral lines of our hypothetical star (19) should be much more affected by the rotation than those of W Ursæ Majoris, and this difference may be even larger if we consider that, according to Stebbins (20), "it seems plausible that there is progressive darkening at the limb in the spectral types ranging from almost nothing in the B stars to extreme effects in classes K and M."

The value of the term sin i is not known unless the star is an eclipsing binary, and it is not possible, in general, to compute the individual values of V_0 for each star. The inclination depends upon the orbital elements in the following manner :—

$$\sin i = \text{const. } K \cdot P^{\frac{1}{3}}(1 - e^2)^{\frac{1}{2}}\frac{(m_1 + m_2)^{\frac{2}{3}}}{m_2} \quad . \quad . \quad (2)$$

If we consider stars of only one spectral class, we may put

$$m_1 + m_2 = \text{const.},$$
$$m_2/m_1 = \text{const.},$$

and consequently

$$\sin i = \text{const. } K \,.\, P^{\frac{1}{3}} \,.\, (1 - e^2)^{\frac{1}{2}} \quad . \quad . \quad . \quad (3)$$

Finally we may assume for spectroscopic binaries, where e is nearly always small,

$$(1 - e^2)^{\frac{1}{2}} = \text{const.}$$

Then

$$\sin i = \text{const. } K \,.\, P^{\frac{1}{3}}.$$

Substituting this in equation (1) we have

$$V_0 = \text{const.} \frac{r \,.\, K}{P^{\frac{2}{3}}} \quad . \quad . \quad . \quad . \quad (4)$$

We see that in a spectroscopic binary the rotational velocity is directly proportional to K and inversely proportional to $P^{\frac{2}{3}}$. Accordingly we may expect that the effects produced by rotation will be particularly strong in binaries with short periods and large amplitudes of their velocity curves. In section 3 we shall show that this is actually the case.

2. *Influence of Rotation upon Line-contour.*—There are a number of factors that influence the width of a spectral line and the distribution of intensities in it (21): (1) Disturbing effects of neighbouring atoms which interfere with the perfect periodicity of the absorbing atom so that its quantum states are not entirely sharp; (2) Doppler effect due to the temperature motion of the particles; (3) Doppler effect due to ascending and descending currents of matter; (4) Doppler effect due to rotation; (5) Compton scattering by free electrons having different velocities; (6) Rayleigh scattering; etc.

The distribution of intensity in a spectral line, $F(\lambda)$, is of importance in our problem. The theory of the contour of spectral lines has been developed chiefly through the researches of Stewart (22) and of Unsöld (23), based upon the remarkable pioneering work of Schuster (24). The influence of the spectrograph itself must also be considered. Wadsworth and others have shown that this factor produces a distribution which may be represented by an exponential function. We attempted to use the following expression for the distribution of intensity in a spectral line as produced by the combination of all factors:—

$$I = F(\lambda) = I_0 e^{-\kappa^2(\lambda - \lambda_0)^2} \quad . \quad . \quad . \quad (5)$$

I_0 and λ_0 are the intensity and wave-length respectively, corresponding to the middle of the line.

Spectrograms made with one-prism and three-prism spectrographs attached to the 40-inch Simeis reflector and analysed by means of a self-registering Koch microphotometer show that the lines occasionally deviate considerably from this expression, and that the form of the intensity distribution is not the same for all lines. This may also be

derived from the observational data relating to the contours of lines, of von Klüber, Unsöld, and Miss Payne (25).

Rotation produces a symmetrical widening of the lines, if the figure of the star is symmetrical with respect to the axis of rotation. Let us suppose that the star is spherical in shape and that its axis of rotation is perpendicular to the line of sight. The rectangular co-ordinates on the apparent disc of the star are connected with the spherical co-ordinates in the following way :—

$$\left.\begin{aligned} x &= \cos b \,.\, \sin l \\ y &= \sin b \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (6)$$

The radial component of the rotational velocity is

$$v = \cos b \sin l \quad . \quad . \quad . \quad . \quad (7)$$

so that all points on the disc having identical values of x have also identical components of rotational velocity.

Consider a star without any darkening at the limb, and let the original distribution of intensity be given by (5). For a rotating star we integrate (5) over the whole disc. For an arbitrary point the middle of the line is given by

$$\lambda_0' = \lambda_0 + \lambda_0 \frac{v}{c},$$

where

$$v = v_0 \cos b \sin l \quad . \quad . \quad . \quad . \quad (8)$$

Since v is numerically (in terms of its maximum value) equivalent to the distance x on the stellar disc from the axis of rotation, we can write for the distribution ot intensity in the line of the integrated spectrum,

$$I_1' = I_0' \int_{-1}^{+1} \int_0^y e^{-\kappa^2 \left[\lambda - \lambda_0 \left(1 \pm \frac{v}{c}\right)\right]^2} dv dy \quad . \quad . \quad (9)$$

Since

$$y = \sqrt{1 - v^2}$$

we may write in place of (9),

$$I_1' = I_0' \int_{-1}^{+1} \sqrt{1 - v^2}\, e^{-\kappa^2 \left[\lambda - \lambda_0 \left(1 \pm \frac{v}{c}\right)\right]^2} dv \quad . \quad . \quad (10)$$

Equation (10) can be integrated mechanically.

For our numerical computations we substituted in (10), in place of the expression given in (5), the distribution $F(\lambda)$ actually recorded by a Koch microphotometer in lines of the Moon's spectrum. While this intensity curve may not hold true for all lines in stellar spectra, it is probably more reliable than equation (5). Since we are concerned at present only with the general aspect of rotational effects, we can safely neglect the small differences in $F(\lambda)$ for individual lines.

The influence of the darkening at the limb, as will be seen below, is small. In one of the numerical examples we have used a darkening coefficient equal to 0·75 (see fig. 1 ; the intensity curve for Y Cygni,

initial line-width 2 Å., drawn between the two curves for the rotating and non-rotating star, represents the solution with darkening at the limb).

The resulting curves will also be influenced by the character of the lines, from the point of view of their energy level. The light coming from the edges of the disc traverses thinner layers of the star's atmosphere and the line originates in higher and cooler regions. Accordingly certain lines must appear strengthened near the edges, while other lines are weakened and perhaps even reversed. It is also clear that the wings of certain lines may disappear near the edges. Obviously

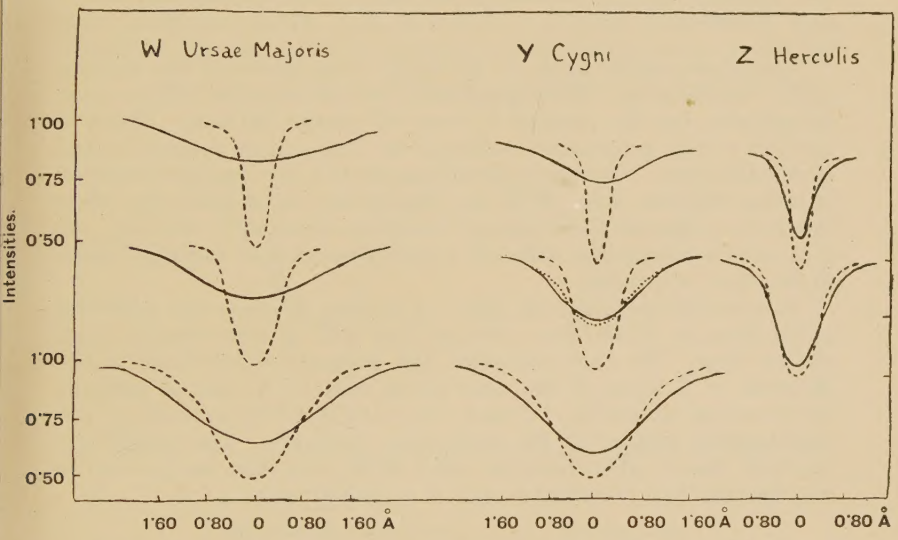

FIG. 1.—Contour of spectral lines, for initial widths of 1, 2, and 4 Å. The initial intensity of the middle of the lines was assumed to be 0·50.

- - - - - Non-rotating stars. —— Effect of rotation. . . . . Solution with darkening.

the low-temperature lines will be the most sensitive ones with respect to rotation.

To illustrate the order of magnitude to be expected for the rotational effect we have made computations for a number of spectroscopic binaries under the assumption that the periods of rotation and of revolution are equal. The effect will, of course, depend upon the initial width and upon the central intensity of the line. We have carried through the computations for lines of 1, 2, and 4 Å. width. The greater values are encountered more frequently in the earlier spectral types, and it is in these that fast rotations are especially probable.

The results are shown in fig. 1 and in Table I. We assume that the initial intensity of the line is $0^{m}\cdot75$, this representing the difference between the continuous spectrum and the centre of the line.

TABLE I.

Central Intensity of Lines for Rotating Stars.
(The initial intensity was assumed to be $0^m.75$.)

Star.	Width.		
	1 Å.	2 Å.	4 Å.
	m	m	m
W Ursæ Majoris	0·12	0·24	0·43
Y Cygni	·17	·35	·52
Z Herculis	·52	·69	
u Herculis	..	·23	

The above results apply to the larger components of the binaries under consideration. Photographically the phenomenon will be more complicated, but the essential features will remain the same. There is not only a very pronounced widening but also a large decrease in the depth of the line. This latter should constitute a very sensitive criterion of stellar rotation (26). With our present-day instruments the effect should be noticeable for rotational velocities exceeding 30 km./sec. It is of interest to note that wide and narrow lines react in varying degrees to the effect of rotation.

It is readily seen that the effect of rotation depends also upon the initial intensity of the line. Deeper lines will be more affected than shallow ones. We have computed the contours of two lines in the spectrum of Y Cygni of the same initial width, 2 Å., but of different initial central intensities, 0·50 and 0·20, in units of the intensity of the neighbouring portion of the continuous spectrum. The results are shown in fig. 2. The intensity-ratio of the two lines in question is not the same in a non-rotating and in a rotating star :

	Intensity-ratio.
For a non-rotating star	2·50
For a rotating star	1·29

Owing to rotation the unequal initial depths become more nearly uniform. This criterion is perhaps less applicable to spectroscopic binaries since the superposition of the continuous spectrum of the secondary star leads to a similar effect. However, in single stars, or in binaries where there is no interference on the part of the fainter component, this difficulty does not exist.

In actual practice the effect of rotation can be studied in two different ways. Consider a single spectral line. Its width and central intensity are functions of many factors, rotation being one of them. In a given number of stars there will be various intensities and line-widths, and it is not *a priori* possible to distinguish the influence of rotation from that of the other factors. We may, however, subdivide our stars into a number of groups, such that the expected rotations differ systematically from one group to the next, while the influences of the other factors remain, on the average, constant. In that case

rotation will stand out and can be investigated separately. This method is purely statistical, and we have followed it in section 3.

Since stellar spectra contain usually many lines, wide and narrow, intense and weak, it should, theoretically, be possible to investigate the effect of rotation for each star individually. Suppose we know, from the solar spectrum, the initial intensities and widths of certain lines. If the spectrum of a star of the solar type shows the narrow lines appreciably too weak, while the wide lines remain about the same, we may conclude that the weakening is due to rotation (provided, of course, that the particular line is not sensitive to changes in absolute magnitude).

An entirely different method can be used in eclipsing variables. We have mentioned before that the effect of rotation has actually been measured from the small variations in radial velocity at the epochs just

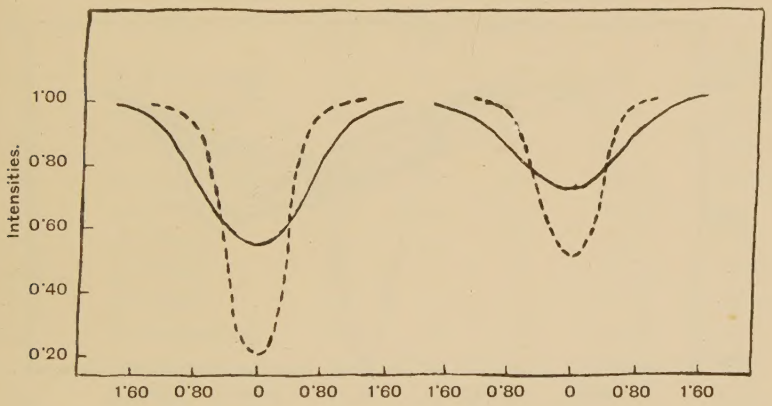

FIG. 2.—Line-contours for Y Cygni. The initial intensities are 0·20 and 0·50.
- - - - - Non-rotating star. —— Rotating star.

preceding and following the middle of eclipse. The line-contours observed during the same epochs will also be subject to rotational changes. The intensity distribution will be asymmetric and the degree of asymmetry will change with the phase. It should also be noted that during eclipse the central depths of the lines are less in a rapidly rotating star than in one having slow rotation. We have computed the contours of a line of the primary component of Y Cygni, for various phases (see fig. 3). We have assumed that (1) the star is spherical, (2) there is no darkening at the limb, (3) the spectrum of the principal star is not affected by that of the fainter. Our last assumption is invalid in the case of Y Cygni, so that, strictly speaking, our computations do not refer to that star. There are, however, many other eclipsing variables that sufficiently closely resemble this hypothetical object. The solution was obtained by a mechanical quadrature of equation (10) with the substitution of our empirical function $F(\lambda)$, as explained above. The computations refer to a line of initial intensity $0^{m}75$. Table II. shows the change in the central depth of the line as a function of phase.

TABLE II.

Central Intensity of a Line of Initial Width 1 Å. *and Intensity* $0^m.75$ *during Eclipse for Y Cygni.*

Phase.	Intensity.
d	m
0·129	0·33
·110	·29
·089	·26
·070	·24
·052	·22
·018	·18

(The phases are counted from the middle of the eclipse.)

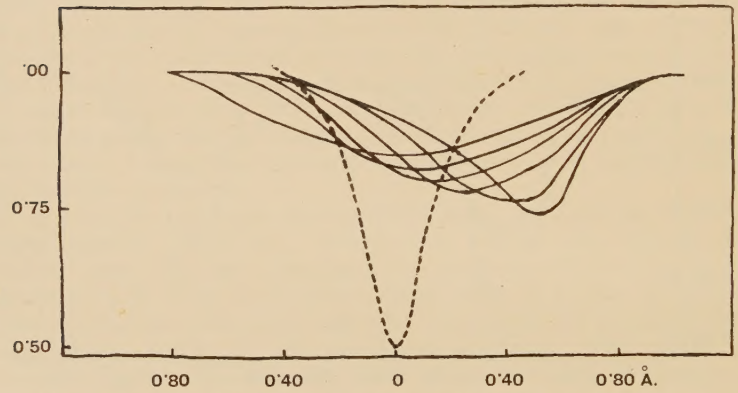

FIG. 3.—Line-contours at different phases during eclipse, for Y Cygni.
- - - - - Non-rotating star.

Investigations, made during an eclipse, of the line-contours in eclipsing variables would doubtless give useful results for the further study of their rotation.

3. *Observational Results for Spectroscopic Binaries.*—We have shown theoretically in section (1) that the rotational effects, if they are at all measurable, should predominate in binaries with short periods and large velocity amplitudes. We shall now show that this is actually the case, and that indeed the effects are so pronounced that they can easily be recognized in a statistical discussion.

Table III. contains the line-widths for a number of B and A type spectroscopic binaries for which spectrograms were available at the Yerkes Observatory. For the B stars the widths of He $\lambda 4472$ and Mg^+ $\lambda 4481$ have been measured. The results agree reasonably well, showing that for these two lines the widening is frequently due to a common cause. The hydrogen lines are, of course, unsuitable for such a study, since their width is quite abnormal. In the A stars only Mg^+ $\lambda 4481$ has been measured.

The probable error of the measures was not determined. But in a previous series it was found to be equal to ± 0·12 Å. from measures of the same plate. It is believed that the line-widths given in Table III. are consistent within about 0·2 Å.

From the resolving power of the Yerkes spectrograph (97,600 for three prisms) we have computed that the minimum width imposed by the properties of the spectrograph is about 0·68 Å. (for one prism). Only line-widths greater than this value have a real physical meaning.

TABLE III.

Measures of Line-widths in Spectroscopic Binaries of Types B and O.

Boss No.	Name.	Width in Å. λ 4472.	Width in Å. λ 4481.	Qual.	P. d	K. Km./sec.
123	29 π Andromedæ	1·6	1·2	g	143·67	47·66
159	23 Cassiopeiæ	1·3	0·7	g	33·75	16·32
425	46 ω Cassiopeiæ	1·9	1·0	g	69·92	29·64
641	42 π Arietis	2·0	1·9	g	3·854	24·77
816	19 τ^5 Eridani	1·0	1·0	g	6·2236	107
913	46 ξ Persei	4·1	..	g	6·951	7·87
920	35 λ Tauri	2·6	..	f	3·953	56·18
1107	94 τ Tauri	2·7	1·7	g	1·505	44·34
1139	9 α Camelopard.	2·7	..	g	7·996	9·0
1147	3 π^4 Orionis	1·3	1·3	g	9·519	25·93
1159	8 π^5 Orionis	2·5	2·7	g	3·700	57·88
1213	66 Eridani	..	1·3	g	5·522	97
1250	19 β Orionis	1·0	0·8	g	21·90	3·771
1301	28 η Orionis	2·0	1·7	g	7·990	144·75
1314	30 ψ Orionis	3·3	2·4	g	2·526	144·12
1333	25 χ Aurigæ	1·1	1·3	g	655·16	20·53
1339	34 δ Orionis	3·2	2·4	g	5·7325	100·12
1353	37 φ^1 Orionis	1·4	1·4	g	3066	13·3
1365	43 θ^2 Orionis	3·3	1·8	g	21·029	93·7
1366	44 ι Orionis	2·2	..	g	29·136	109·90
1388	125 Tauri	1·6	1·7	g	27·864	25·5
1399	−1° 1004 Orion.	2·6	1·6	g	27·160	93·04
1525	67 ν Orionis	1·2	0·9	g	131·26	34·09
1872	27 Can. Maj.	2·2	..	g	121	25
1899	29 Can. Maj.	3·8	..	g	4·393	218·44
1901	30 τ Can. Maj.	2·8	..	g	154·80	52·1
2445	76 $\varkappa$ Cancri	1·3	0·8	g	6·393	67·8
3138	31 Corvi	3·1	2·2	g	1·503	118·19

TABLE III.—*continued.*

Boss No.	Name.	Width in Å. λ 4472.	Width in Å. λ 4481.	Qual.	P. d	K. Km./sec.
3281	5 $\varkappa$ Draconis	2·1	2·1	g	0·89038	18·9
3476	67 α Virginis	3·0	..	g	4·014	126·1
3988	7 ζ Coronæ	2·9	1·7	g	12·585	134·82
4008	25 A Serpentis	2·7	1·6	g	38·95	50·52
4062	6 π Scorpii	3·8	2·9	g	1·571	138
4388	68 u Herculis	3·6	3·2	g	2·051	99·50
4604	13 μ Sagittarii	1·1	1·2	g	180·2	64·5
4779	112 Herculis	1·9	1·4	f	6·3624	18·0
4794	11 δ^1 Lyræ	2·8	1·8	g	88·112	33·68
4864	18 Y Aquilæ	1·3	1·5	g	1·302	27·59
4934	46 v Sagittarii	1·4	0·8	g	137·937	48·15
5018	44 σ Aquilæ	2·5	..	g	1·950	163·52
5375	57 Cygni	1·8	1·1	g	2·855	110·4
5532	8 β Cephei	1·5	..	g	0·190	16·9
5565	56° 2617 Cephei	2·0	..	p	1·364	77·01
5764	2 Lacertæ	1·4	1·1	g	2·616	80·3
5913	16 Lacertæ	1·2	1·2	g	12·3106	22·2
6046	1 Hev. Cassiopeiæ	1·6	1·3	g	6·067	59·06

Measures of Line-widths in Spectroscopic Binaries of Type A.

Boss No.	Name.	Width λ 4481.	Qual.	P. d	K. Km./sec.
10	21 α Andromedæ	1·1	f	96·67	30·75
82	+43° 92 Andromedæ	0·8	g	3·956	41·7
145	20 π Cassiop.	1·3	f	1·964	117·32
428	6 β Arietis	1·4	g	107·0	32·6
923	36 τ Eridani	0·8	g	0·85347	35·7
986	b^1 Persei	1·7	g	1·527	41·89
1244	14 Aurigæ	1·1	f	3·789	21·56
1452	31 Camelop.	1·6	g	2·933	76·02
1457	136 Tauri	1·1	g	5·969	48·9
1492	2 Monocerotis	0·9	g	9·355	57·1
1501	61 μ Orionis	0·6	g	4·447	30·8
1515	40 Aurigæ	1·1	f	28·28	51·38
1690	24 γ Geminorum	0·9	g	2175	6·12
1979	br α Geminorum	0·7	g	9·219	13·56
1979	ft α Geminorum	1·0	g	2·928	31·76

TABLE III.—*continued.*

Boss No.	Name.	Width λ 4481.	Qual.	P. d	K. Km./sec.
2754	30 H Ursæ Majoris	1·0	g	11·583	34·07
2900	45 ω Ursæ Majoris	1·0	g	15·840	20·64
2987	55 Ursæ Majoris	1·4	g	2·5	38·5
3123	95 o Leonis	1·0	g	6·6	65
3182	+78° 412 Draconis	2·2	g	1·271	63·2
3210	15 η Virginis	0·7	g	71·9	26·8
3323	32 d^2 Virginis	1·3	f	38·3	40·99
3363	77 ε Ursæ Majoris	1·1	g	1515	3·5
3371	12 α^2 Can. Ven.	0·8	g	5·50	21·5
3474	79 ζ Ursæ Majoris	0·9	g	20·536	69·22
3626	11 α Draconis	0·8	g	51·38	46·25
3825	19 δ Libræ	1·7	g	2·327	76·5
3961	5 α Coronæ	2·0	g	17·36	34·93
4328	58 ε Herculis	1·6	g	4·024	70·39
4462	55 ξ Serpentis	0·8	g	2·292	19·35
4752	6 ζ Lyræ br.	1·2	g	4·300	51·24
4788	50 Draconis	1·3	g	4·118	79·12
5099	61 φ Aquilæ	0·8	g	3·320	38·25
5171	65 θ Aquilæ	0·9	f	17·124	46·0
5469	67 σ Cygni	1·1	g	11·043	1·98
5579	77 Cygni	1·4	p	1·729	109·7
5600	49 δ Capricorni	1·5	g	1·023	65·67

The results of the measurements are plotted in figs. 4 and 5. The abscissæ are expressed in log (10 P) and the ordinates represent the respective values of K. The stars occupy an area (27) limited at the bottom by the x-axis, at the left by a line that seems to be roughly straight and vertical, and at the top by a curve of the equation $K = C \cdot P^{-\frac{1}{4}}$. The conditions for finding a strong rotational widening of the lines (large K and small P) are fulfilled in the upper left portion of the occupied area. A glance at the figure shows that it is in these parts of the diagram that only wide lines are observed (28).

It is interesting to note that a few wide-line stars have large P and small K. If widening of spectral lines were due to rotation only this would indicate that the components of long-period spectroscopic binaries may occasionally have a fast rotation. But it is, of course, quite possible that the broadening depends on other factors. We conclude that the location of a spectroscopic binary in the upper left corner of the diagrams is a sufficient condition for observing wide lines in their spectra. However, if a star is located to the right or to the bottom of the diagrams the lines are not necessarily narrow.

The predominance of fast rotational velocities in short-period spectroscopic binaries can be brought out also in a different way. It is a well-known fact that stellar spectra of the same type greatly differ from one another in the appearance of their lines. This is especially true in the earliest spectral types, for which Adams and Joy have introduced the designations " sharp " and " nebulous." The character of spectrum is, as Adams and Joy have shown, correlated with absolute magnitude. It is probable that a large portion of this effect is due to atmospheric conditions within the stars. However, if we consider only

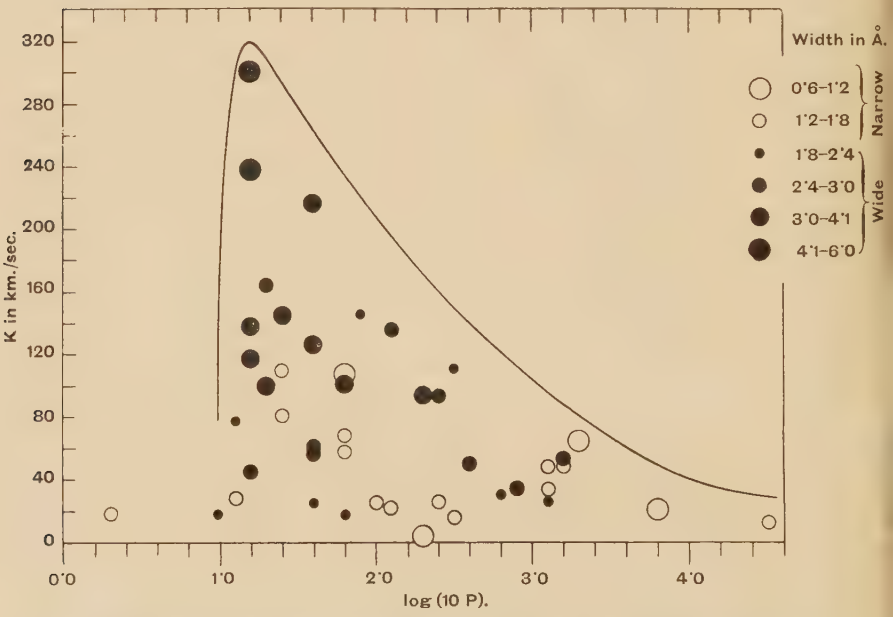

FIG. 4.—Line widths in spectroscopic binaries of types O and B. The two highest stars in the diagram are V Puppis and μ^1 Scorpii, for which Miss Maury finds a width of 5·8 Å and 4·9 Å respectively.

spectroscopic binaries an interesting detail is brought to light. We would naturally expect that rapid rotation would produce " nebulous " lines. In the following table we have used the spectroscopic binaries of Moore's Third Catalogue (29), for which the subdivision " n " or " s " is known. A few additional stars were taken from Beer's recent paper on spectroscopic binaries (30).

In agreement with our expectation, short periods show a distinct tendency to be associated with " nebulous " lines. It might be noted that for spectroscopic binaries of type B8–F2 the relative number of " s " is much greater than for single stars. But this may be due to selection, since it is easier to measure sharp-lined stars than those having diffuse lines.

TABLE IV.

Spectroscopic Binaries arranged according to Period and Character of Spectrum.

Spectrum.	Period.	"n."	"s."
Bo–B6	Longer than 3^d	20	10
,,	Shorter ,, ,,	8	0
B8–F2	Longer than 3^d	9	28
,,	Shorter ,, ,,	8	4
Bo–F2	Longer than 3^d	29	38
,,	Shorter ,, ,,	16	4

The existence of fast rotation in spectroscopic binaries of short period seems thus to be established beyond any doubt. The line-

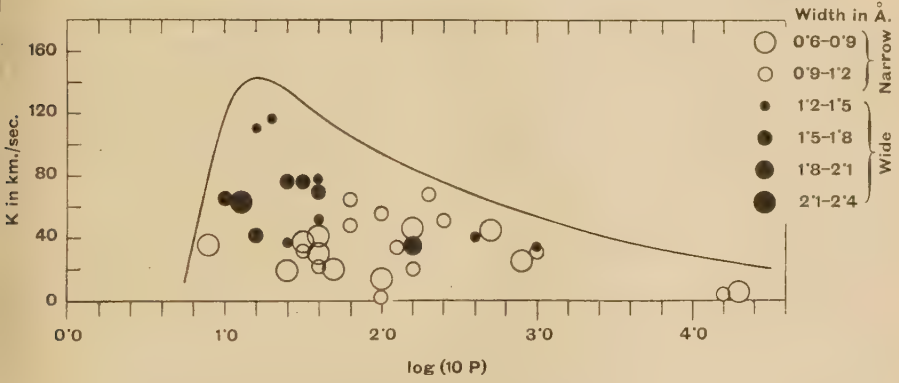

FIG. 5.—Line widths in spectroscopic binaries of type A.

widths in Table III. are strongly correlated with central intensity, so that the effect of widening is associated with one of decrease in the residual intensity of the lines, as is demanded by the theory of section 2. We hope in future to apply some of the differential methods of section 2 to individual stars, in order to get more quantitative results regarding the actual speeds of rotation.

4. *The Rotational Effect in the Spectrum of Jupiter.*—To find what might actually be expected in the case of a rotating star, spectrograms of Jupiter were obtained in 1927 by means of a single-prism spectrograph attached to the 40-inch Simeis reflector. The rotation of Jupiter causes in the reflected sunlight a Doppler shift of about 25 km./sec. at the equatorial limb. The experiment consisted in comparing two spectrograms of Jupiter. The one was taken with the slit perpendicular to the equator and with the image of the planet accurately bisected by the slit. In this case there was, of course, no effect of rotational widening nor any decrease in line-depth. The next plate was taken with approximately the same exposure time, the image of the planet

being oscillated uniformly across the slit by means of an electric motor for very slow motion, thus giving an integrated spectrum for the whole planet. This is probably a fair approximation to the spectrum of a rotating star, in spite of the notoriously strong darkening of Jupiter at the limb. The exposures, usually made near the meridian, were about 25 minutes long. The linear dispersion in the region measured was about 12·5 Å. per mm. The total number of plates was ten. The spectrograms were run through a Koch microphotometer with gear ratio 48·5 to 1.

The widths of a number of lines were measured on the microphotometer tracings (see Tables V. and VI.). The results are affected by blending with other lines so that the precision is not great. However, since we are here concerned only with differential effects, these uncertainties, as well as instrumental or other errors, are of no consequence.

There is a slight widening of the spectral lines in the integrated spectrum, but its amount is of the same order of magnitude as the mean error, and is therefore not certain. Evidently an equatorial velocity of rotation of 25 km./sec. produces an amount of widening that is at the very limit of what the measurements can give. The rotation should probably be appreciably faster, perhaps of the order of 50 to 100 km./sec., to produce effects upon the line-widths that are definitely measurable.

The measures of line-intensities give very much better results. Since the exposure times of the spectrograms were such that the photographic densities of the continuous spectrum are the same, we have measured the depths of the lines in mm.'s and summarised in Table VII. the differences for each line.

The results show that the lines of the integrated spectrum are appreciably shallower than those which are not affected by rotation, but this difference is not very large and is noticeable only after a careful study of the material.

We may thus conclude that the effect produced by a rotational velocity of the order of 25 km./sec. is about at the limit of what the measurements can reveal. Since in actual practice we shall never be able to compare with each other two spectra that are as much alike as two plates of Jupiter, the lower limit should be taken rather above 25 km./sec. However, there is every reason to believe that rotational velocities exceeding 50 km./sec. should be measurable without much difficulty. That such velocities are not infrequent, is shown by our statistical discussion of spectroscopic binaries. Whether or not single stars also frequently rotate with such high velocities remains to be seen.

TABLE V.

The Widths of Spectral Lines.

	λ 4030·9.			λ 4033·2.		
	Normal.	Integr.	Diff.	Normal.	Integr.	Diff.
1927	mm.	mm.		mm.	mm.	
Oct. 10	0·188	0·190	+0·002	0·140	0·136	−0·004
19	·161	·184	+ ·023	·138	·130	− ·008
24	·159	·173	+ ·014	·126	·140	+ ·014
26	·147	·157	+ ·009	·118	·107	− ·011
28	0·173	0·157	−0·016	0·124	0·132	+0·008
		Mean	+0·006±0·0066 (m. e.)			0·000±0·0048

	λ 4034·6.			λ 4035·8.		
	Normal.	Integr.	Diff.	Normal.	Integr.	Diff.
1927	mm.	mm.		mm.	mm.	
Oct. 10	0·099	0·095	−0·004	0·082	0·091	+0·009
19	·118	·103	− ·015	..	..	..
24	·122	·120	− ·002	..	..	..
26	·109	·122	+ ·013	..	..	..
28	0·099	0·089	−0·010	0·122	0·122	0·000
			−0·004±0·0047			+0·004±0·0045

	λ 4052·0			λ 4055·0.		
	Normal.	Integr.	Diff.	Normal.	Integr.	Diff.
1927	mm.	mm.		mm.	mm.	
Oct. 10	0·194	0·184	−0·010	..	..	..
19	·181	·177	− ·004	..	..	..
24	..	..	..	..	..	..
26	·188	·198	+ ·010	0·208	0·212	+0·004
28	0·188	0·179	−0·009	0·165	0·171	+0·006
			−0·003±0·0046			+0·005±0·001

	λ 4063·6.			λ 4071·9.		
	Normal.	Integr.	Diff.	Normal.	Integr.	Diff.
1927	mm.	mm.		mm.	mm.	
Oct. 10	0·330	0·318	−0·012	0·293	0·297	+0·004
19	·217	·252	+ ·035	·247	·268	+ ·021
24	·268	·282	+ ·014	·268	·268	·000
26	0·305	0·305	0·000	0·289	0·289	·000
28	..	..	..	0·247	0·250	+0·003
			+0·009±0·010			+0·006±0·0039

TABLE VI.

Summary of Line-widths.

λ.	Integr.—normal.	Mean Error.	λ.	Integr.—normal.	Mean Error.
4030·9	+0·006	±0·0066	4052·0	− ·003	·0046
4033·2	·000	·0048	4055·0	+ ·005	·0010
4034·6	− ·004	·0047	4063·6	+ ·009	·0100
4035·8	+ ·004	·0045	4071·9	+ ·006	·0039

TABLE VII.

Intensity-differences of Lines.

Normal—Integrated.

λ.	Oct. 10.	Oct. 19.	Oct. 24.	Oct. 26.	Oct. 28.
4030·9	+2·0	+2·2	+0·5	+1·5	+0·8
4033·2	+1·5	−0·3	+1·5	+1·0	+0·5
4034·6	0·0	−0·6	+0·5	0·0	0·0
4035·8	+0·2	+0·4	+0·3	−0·5	..
4037·2	+0·4	+0·5	+0·1	−0·5	0·0
4038·9	+0·7	−0·4	+0·6	+0·8	0·0
4041·6	+0·8	−1·4	+0·2	..	0·0
4044·2	+2·8	+1·5	−0·6	+0·5	+1·0
4045·8	0·0	−0·6	+1·5	0·0	+1·7
4048·8	+0·1	−0·5	−0·3	+2·0	+0·5
4050·8	..	0·0	..	..	..
4052·0	..	−0·8	..	+1·7	+0·3
4055·0	..	..	..	+0·5	+0·4
4057·6	..	..	..	+1·2	..
4063·6	..	+0·9	+2·5	+1·5	+1·0
4067·1	+3·8	+0·7	+0·2	+0·9	+0·7
4071·9	+1·0	+1·0	+0·4	+2·2	+1·1
4074·9	+2·0	+0·5	+0·2	+1·0	+1·0
4076·9	..	+0·5	+2·0	−0·5	+1·3
4077·9	..	+0·7	+1·1	−0·5	..
4078·4	..	−0·2	0·0	−0·2	−0·4
Mean	+1·18±0·33	+0·22±0·19	+0·63±0·22	+0·66±0·21	+0·58±0·14

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(15) *Ap. J.*, **49**, 189, 1919.
(16) *Publ. A.S.P.*, **39**, 160, 1927.
(17) It has previously been shown that all real double stars of spectral type B have periods longer than $1^d \cdot 3$ (*M.N.*, **86**, 63, 1925; *Ap. J.*, **66**, 117, 1927).
(18) *Publ. A.S.P.*, **32**, 315, 1920.
(19) The eclipsing variable V Puppis (Sp. B1p; P = $1^d \cdot 454$; K = 302 km./sec.) closely resembles our hypothetical B star. According to Hellerich (*loc. cit.*) the rotational velocity for the brighter component of this star is 276 km./sec., and the corresponding line-width should be 7·9 Å. Miss Maury (*Harvard Annals*, **84**, 186, 1920) has measured the widths of several lines in this star and has found a mean of 5·78 Å.—an unusually high value for a B-type star.
(20) *Ap. J.*, **54**, 91, 1921.
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(26) The numerical results cannot be directly compared with those of Carroll, since he uses spectral lines of zero intensity, whereas our lines are taken from observational data, and their depths are very moderate
(27) *M.N.*, **86**, 63, 1925.
(28) In fig. 4 the stars α Camelopardalis and ξ Persei, the orbits of which are based on the detached calcium lines, have been excluded. These stars are peculiar and their velocity curves may not be due to orbital motion. Both have wide lines and small values of K. They would, of course, in no way contradict the above statement, even if they had been included. On the other hand, we have included the two stars V Puppis and μ^1 Scorpii, based on Miss Maury's measurements.
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(30) *Veröff. der Berlin-Babelsberg Sternwarte*, Band v., Heft 6.

September 1928.

Shortt Clocks and the Earth's Rotation. By J. Jackson, M.A., D.Sc.

(Communicated by the Astronomer Royal.)

In *M.N.*, **88**, 465, a description was given of the performance of the clocks Shortt 3 and Shortt 11 from the time of their installation, 1924 November and 1926 May respectively, till the end of 1927. It was there shown that the going of these clocks over long intervals of time far surpassed that of earlier types of clock. The accuracy of the clocks indeed was such that it was found advisable to allow for the non-uniformity of sidereal time resulting from nutation—the principal term of coefficient $1^s \cdot 06$ and period 18·6 years indeed varies so slowly that it causes no trouble, but the term of coefficient $^s \cdot 08$ and six-month period

is large enough and varies rapidly enough to be considered. The correction required to the clock time to reduce it to true sidereal time was therefore written in the form

$$\alpha + \beta t + \gamma t^2 + \delta\int(\mathrm{T} - \mathrm{T}_0)dt + \text{Nutation}.$$

The coefficient γ was found to be nearly steady throughout 1927, with a value of $1^s{\cdot}84$ when t was measured in units of 100 days for Shortt 3; the value for Shortt 11 appeared to be of a similar amount, but to be gradually decreasing. The temperature coefficient, though small, was clearly defined of amount $^s{\cdot}003$ per day per 1° F. For Shortt 3, the correction was given in formula (3), p. 469, as

$$7^s{\cdot}775 + 8^s{\cdot}3636t + 1^s{\cdot}8400t^2 + 0^s{\cdot}298\int(\mathrm{T} - 54°{\cdot}8)dt + \text{Nutation}$$

where t is measured in units of 100 days from 1927 December $1^d\,0^h$ G.C.T.

The following table is a continuation of Table III. on p. 469, of vol. lxxxviii., and gives a comparison of the time determinations made with the reversible transit B in 1928 with the above formula which was deduced from observations made from 1927 March to December. The observations for the first half of the year were made by myself and for the second half by Mr. Finch. Our relative personal equation is negligible. It should be recalled that the formula satisfied the observations exactly on 1926 July 19, but left a residual of $0^s{\cdot}76$ in October of that year. It satisfied the observations from 1927 March till December to $0^s{\cdot}1$. Comparison with formula (3)′, which does not materially differ from (3), is not given here. The last column shows the number of stars observed south and north of the zenith.

Table I.

Comparison of Clock Errors observed with Transit B with those predicted in M.N., **88**, 469, *from Observations made from March to December* 1927. (*Multiples of* 10^s *have been dropped as they arise in the 3rd and 4th columns.*)

1928.	Temperature Integral.	Formula (3).	Obs.	O−C$_s$.	No. of Stars.
	s	s	s	s	
Jan. 6	+ ·036	+0·180	+0·176	−·004	4, 2
19	·051	1·488	1·482	−·006	6, 4
Feb. 1	·060	2·869	2·838	−·031	6, 4
13	·071	4·180	4·102	−·078	6, 4
20	·078	4·994	4·897	−·097	5, 3
28	·085	5·891	5·804	−·087	3, 3
Mar. 15	·074	7·773	7·746	−·027	4, 5
17	·073	8·022	7·956	−·066	3, 3
28	·074	9·376	9·252	−·124	3, 3
30	·077	9·669	9·529	−·140	4, 2
Apr. 2	·082	0·027	9·832	−·195	4, 3
18	·085	2·107	1·808	−·299	5, 6

TABLE I.—*continued.*

1928.	Temperature Integral.	Formula (3).	Obs.	O−C$_3$.	No. of Stars.
	s	s	s	s	
Apr. 24	·078	2·919	2·582	−·337	7, 5
28	·079	3·490	3·164	−·326	3, 3
May 9	·090	5·027	4·694	−·333	6, 5
18	·091	6·321	6·023	−·298	7, 4
24	·084	7·241	6·929	−·312	6, 4
30	·083	8·121	7·849	−·272	4, 3
June 6	·105	9·235	9·040	−·195	3, 2
11	·116	0·060	9·896	−·164	4, 3
15	·122	0·652	0·491	−·161	5, 3
22	·120	1·832	1·745	−·087	5, 4
27	·119	2·596	2·530	−·066	2, 1
July 1	·123	3·262	3·289	+·027	4, 3
7	·134	4·275	4·295	+·020	3, 1
11	·150	4·944	4·996	+·052	5, 1
13	·159	5·298	5·351	+·053	5, 1
17	·186	6·029	6·096	+·067	3, 2
21	·229	6·747	6·836	+·089	6, 4
25	·282	7·480	7·562	+·082	6, 5
Aug. 2	·425	9·020	9·045	+·025	3, 1
8	·533	0·153	0·049	−·104	6, 4
9	·549	0·349	0·303	−·046	3, 2
13	·615	1·147	1·068	−·079	3, 3
17	·684	1·921	1·742	−·179	4, 3
20	·738	2·502	2·292	−·210	5, 2
27	·867	3·907	3·621	−·286	5, 4
Sept. 1	·962	4·916	4·605	−·311	6, 5
4	1·016	5·502	5·160	−·342	5, 3
7	1·072	6·123	5·754	−·369	5, 4
11	1·146	6·968	6·607	−·361	5, 3
18	1·274	8·371	7·954	−·417	7, 3
25	1·398	9·881	9·382	−·499	5, 3
30	1·472	0·867	0·336	−·531	5, 2
Oct. 4	1·532	1·721	1·181	−·540	6, 3
9	1·603	2·786	2·218	−·568	3, 3
11	1·638	3·204	2·629	−·575	5, 4
17	1·763	4·552	3·974	−·578	5, 3
21	1·857	5·442	4·908	−·534	5, 3
25	1·951	6·322	5·850	−·472	6, 3
28	2·022	7·011	6·535	−·476	5, 3
Nov. 4	2·188	8·633	8·220	−·413	5, 4
8	2·282	9·563	9·152	−·411	5, 3

TABLE I.—*continued.*

1928.	Temperature Integral.	Formula (3).	Obs.	O−C$_3$.	No. of Stars.
	s	s	s	s	
Nov. 17	2·495	1·682	1·308	−·374	5, 4
23	2·622	3·137	2·732	−·405	5, 3
24	2·646	3·339	2·930	−·409	2, 1
Dec. 4	2·884	5·836	5·447	−·389	2, 1
7	2·956	6·521	6·177	−·344	5, 3
9	3·023	7·023	6·642	−·381	6, 3
14	3·122	8·337	7·993	−·344	5, 4
17	3·192	9·030	8·703	−·327	2, 2
26	3·406	1·304	0·953	−·351	5, 3
29	+3·479	+2·080	+1·734	−·346	4, 3

The residuals which are shown in fig. 1 reached a maximum of 0^s·58 in October, *i.e.* fully nine months later than the last observation used in the formation of formula (3). The turning points in the errors from prediction were

	s
April 24 . .	− ·337
July 21 . .	+ ·089
October 17 . .	− ·578.

The value on December 31 was − ^{s}·346, so that there was a total fluctuation in the year of 1^s·66. This gives an average departure of the predicted from the actual rate, independently of sign, of ^{s}·0046 daily, the greatest being about ^{s}·01 in August and September.

The residuals between the observed and predicted clock errors show oscillations of a kind which clearly cannot be explained by slight adjustment of the constants, but it was thought worth while to recompute them and at the same time those for Shortt 11, which has constants remarkably similar to those for Shortt 3. Consequently the residuals from the old formula for the period 1927 March to 1928 December were grouped together in intervals of somewhat less than a month and analysed. The resulting corrections to formula (3) are :—

Shortt 3.

$$-\,{}^s\cdot064 - {}^s\cdot0108t - {}^s\cdot0144t^2 - {}^s\cdot0158\int(\mathrm{T} - 54^\circ\cdot8)dt.$$

Shortt 11.

$$-\,1^s\cdot007 + {}^s\cdot2761t - {}^s\cdot0858t^2 - {}^s\cdot0456\int(\mathrm{T} - 54^\circ\cdot8)dt$$

where t is computed as previously from 1927 December 1^d 0^h G.C.T.

Applying these to formula (3) above we get the following corrections to the two clocks.

Shortt 3.

$$7^s\cdot711 + 8^s\cdot3528t + 1^s\cdot8256t^2 + 0^s\cdot2822\int(\mathrm{T} - 54^\circ\cdot8)dt + \text{Nutation}.$$

Shortt 11.

$$6^{s}\cdot768 + 8^{s}\cdot6397t + 1^{s}\cdot7542t^{2} + 0^{s}\cdot2524\int(T - 54^{\circ}\cdot8)dt + \text{Nutation}.$$

In these formulæ t is computed from 1927 December 1.

If we bring forward the epoch by 500 days to 1929 April 14, and at

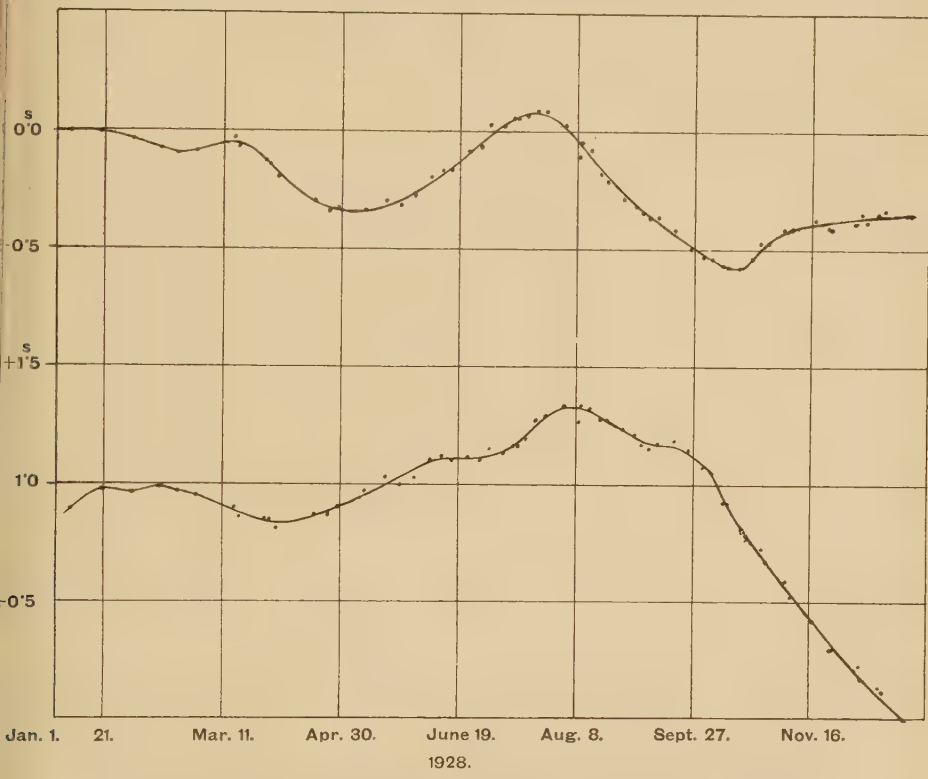

FIG. 1.—Comparison of the corrections to Shortt 3 and Shortt 11 with the formula (3) based on observations 1927 March to December referred to Shortt 3. The interval between the vertical lines is 50 days, between the horizontal lines 0·5 seconds.

the same time alter the standard temperature to 62°·8 F., at which the clock room is now controlled, we obtain the following equations numbered consecutively to those in the previous paper.

Shortt 3.

(5) $100^{s}\cdot797 + 28^{s}\cdot8656t + 1^{s}\cdot8256t^{2} + 0^{s}\cdot2822\int(T - 62^{\circ}\cdot8)dt + \text{Nutation}.$

Shortt 11.

(6) $98^{s}\cdot905 + 28^{s}\cdot2002t + 1^{s}\cdot7542t^{2} + 0^{s}\cdot2524\int(T - 62^{\circ}\cdot8)dt + \text{Nutation}.$

A comparison of these formulæ with the observed values is given in Table II.; the data before 1927 March depend on observations with the large transit circle, and were not used in the analysis.

TABLE II.

Comparison of Clock Corrections for Shortt 3 and Shortt 11 with Formulæ (5) and (6) respectively. (Observed—Computed.)

1926/1927.	No. 3.	No. 11.
	s	s
July 9	+0·41	..
19	+ ·20	..
29	− ·03	..
Aug. 8	− ·20	+7·94
18	− ·29	7·42
28	− ·41	6·87
Sept. 7	− ·47	6·31
17	− ·49	5·87
27	− ·56	5·26
Oct. 7	− ·58	4·72
17	− ·59	4·19
27	− ·58	3·70
Nov. 6	− ·60	3·09
16	− ·58	2·56
26	− ·52	2·18
Dec. 6	− ·44	1·91
16	− ·37	1·69
26	− ·38	1·41
Jan. 5	− ·32	1·13
15	− ·26	0·96
25	− ·21	0·77
Feb. 4	− ·09	0·69
14	− ·05	0·52
24	− ·06	0·36
Mar. 6	+ ·01	0·26
16	+ ·01	+0·12
26	+ ·13	..
Apr. 5	+ ·09	..
15	+ ·03	..
25	+ ·03	..
May 5	−0·05	..

1927/1928.	No. 3.	No. 11.
	s	s
Mar. 19	+0·026	+0·084
May 26	+ ·044	− ·350
June 12	− ·018	− ·354
July 23	− ·090	− ·133
Aug. 16	− ·046	+ ·190
Sept. 12	+ ·026	+ ·452
Oct. 2	− ·041	+ ·520
20	− ·051	+ ·401
Nov. 10	+ ·047	+ ·280
Dec. 12	+ ·070	− ·051
29	+ ·069	− ·161
Jan. 13	+ ·069	− ·152
Feb. 7	+ ·018	− ·151
24	− ·004	− ·189
Mar. 16	+ ·050	− ·292
30	− ·050	− ·350
Apr. 23	− ·207	− ·313
May 17	− ·185	− ·197
June 8	− ·054	− ·101
27	+ ·114	− ·053
July 12	+ ·217	+ ·007
25	+ ·259	+ ·173
Aug. 10	+ ·139	+ ·254
21	+ ·010	+ ·257
Sept. 6	− ·082	+ ·246
24	− ·184	+ ·296
Oct. 10	− ·237	+ ·161
25	− ·135	+ ·045
Nov. 10	− ·004	− ·098
Dec. 2	+ ·067	− ·182
21	+0·154	−0·222

It will be seen that the residuals are very similar to those deduced from formula (3). For Shortt 3 there is a range in the residuals from $+^{s}\cdot 26$ to $-^{s}\cdot 24$ between 1927 March and 1928 December, but when extrapolated backwards there was a residual of $0^{s}\cdot 60$ in 1926 October, and the daily rate in 1926 July was erroneous by $^{s}\cdot 022$. The performance of Shortt 11 was not so good, the range during the interval from 1927 March to 1928 December, was from $+^{s}\cdot 52$ to $-^{s}\cdot 35$, or approximately twice that of Shortt 3, while extrapolation backwards was seriously in error, and in 1926 August the error in rate was $^{s}\cdot 05$ daily.

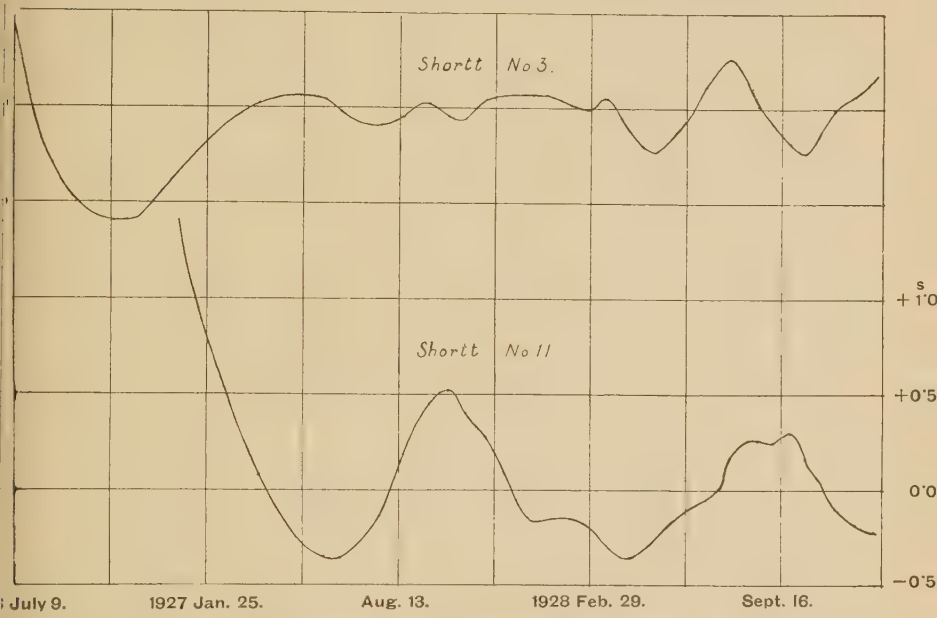

Fig. 2.—Comparison of observed clock corrections with those computed from formulæ (5) and (6). The vertical lines are at intervals of 100 days, and the horizontal lines at intervals of 0·5 seconds.

It will be seen from the diagram showing these residuals (fig. 2) that the ways in which the two clocks depart from the formulæ when extrapolated backwards are very similar. It would indeed be reasonable to expect the clocks to take some time to settle down after being started in 1926 July. Shortt 3 had previously had a run of $1\frac{1}{2}$ years and might have settled down more quickly than Shortt 11, which was only delivered at the observatory in 1926 May.

In considering the actual performance of the Shortt clocks, the form of the formula used must be first examined. For a perfect clock the coefficients γ and δ should be zero. But no mischief is done, apart from decreased weight in the result, if (1) they are accurately determined, (2) they can be accurately applied, which means if the tem-

perature is accurately known, and what is most important (3) their permanency can be guaranteed.

Temperature.—The temperature term should in fact give no insurmountable difficulties. The coefficient δ is comparatively small, and it should be possible, if necessary, to further reduce it. Temperatures can now be controlled so that the variation is small, but no thermostat can act perfectly throughout the whole length of the pendulum. What is required, so far as the clocks at Greenwich are concerned, is to know the temperature, so that for one year compared with another, the error in the temperature should not systematically amount to $0°{\cdot}1$ F.

Variations in temperature may, however, give rise to much more serious troubles than have been considered in this paper. Several excellent clocks of the older patterns have been known to show large variations of daily rate for a difference of 1° C. between the top and bottom of the clock case. Whatever the cause of this may be, it cannot be connected with the escapement. The fact that the free pendulums have copper cases and that they are in rooms where the changes of temperature have been gradual should ensure that the stratification in the clock cases is not very large.

It is, however, desirable that such a possible source of error should be inquired into. The apparatus for experiments is not available at Greenwich, and in any case it is clearly undesirable to interfere at present with the clocks which have already had an undisturbed run of $2\frac{1}{2}$ years. An experiment which might be attempted would be to have two free pendulum clocks which could be compared to something like $^{s}{\cdot}001$ or less, and determine the effect of raising the temperature of the top half of the case above the lower by, say, 1° C. for 24 hours.

The Secular Term.—It will be seen that the formula for the clock correction contains a secular term, which though unimportant for runs of a week or two assumes large dimensions in the course of a year. A variation of this term by only 1 per cent. of its amount would accumulate an error of a quarter of a second in a year. So long as the origin of this term is unknown, it must make conclusions based on the performance of these clocks over a period of years somewhat unconvincing. It has generally been assumed that it is due to a growth of the invar rod. This assumption is based on the known fact that invar grows for some time after its manufacture.

But the secular term in the going of the Shortt clocks would require a growth of the invar amounting to 1μ in 120 days. For the first nine months after Shortt 3 was installed its change of rate was three times its present value, then it suddenly changed to a little more than its present value, and for the last two years it has been practically constant. This in no way resembles the growth of invar generally recorded. Its total amount in 4 years far exceeds what would have been expected, and the uniform rate of the change in the last 2 years does not resemble the irregular slowly decreasing growth of invar. Observations concerning invar too, refer to a bar carefully preserved in a horizontal position. The pendulum rod, with a heavy bob at its lower end, is swinging through a small arc about the vertical, and every 30 seconds

a small impulse with a component along the length is applied. It is therefore by no means certain that the growth of invar is the cause of the secular term.

It is easy to make other suggestions though not so easy to test them. The invar rod is attached to a suspension spring of elinvar. A change of 1μ per 120 days in the point of flexure of the spring could produce the observed effect. The stability of the bob is important. The bobs of these pendulums are made of type metal,* and are cylindrical in shape with a central hole. The bob rests on a small upright tube of invar (No. 3) or brass (No. 11) pinned to the pendulum rod. It is possible to think that the bob is slowly sinking on its support. Chemical changes appear incapable of producing a sensible effect. It is rather extraordinary how in 2 years the pressure inside the case of No. 3 has apparently decreased gradually by about 2 mm., from 36 mm. to 34 mm., according to the mercury gauge, while according to the oil gauge it has decreased by 10 mm. A graduated silver scale which was introduced into No. 11 for the accurate reading of the arc has become unreadable, apparently from the action of mercury vapour. It appears possible that some mercury may have amalgamated with the type metal of the bob. However, if we assume the pendulum to consist of a uniform invar rod of 500 gms. with a bob at the end weighing 5000 gms., deposition of ·01 gm. of mercury on the bob would only alter the daily rate by ^{s}·001, whereas the rate has actually altered in 2 years by ^{s}·27. There appears to be no reason why in future type metal should be used for the bob. The chief reason for its use appears to be its high density, but this is of no great importance where the pressure is very low. With the temperature accurately controlled it might also be worth trying a steel instead of an elinvar suspension spring.

A cause of the remaining irregularities in the clock may be irregular growth of the invar rod. Professor Boys advocates the use of fused quartz which has a very low temperature coefficient, and for which he believes that the mechanical difficulties of connecting the suspension spring, bob, and impulse wheel are surmountable.

The idea that the performance of the Cottingham clock might throw some light on the secular term led to an analysis of its errors. This clock has a pendulum quite similar to the free pendulums as regards the materials of its construction. The performance, however, has been rather spoilt by the fact that the temperature coefficient is large, about ^{s}·04 per day per 1° F. An analysis of its performance showed a secular term two-thirds that of the Shortt clocks, but in the opposite direction. Clocks in general tend to gain with age as the oil thickens, and it is quite likely that the secular term of the Cottingham clock is the result of an effect of this kind, so that it does not shed any light on the Shortt clocks which have practically no oil except a trace at the pivot of the wheel which receives the impulse, and the axes of the impulsing mechanism, although the latter are not connected with the pendulum in the same way as in an escapement clock with the usual train of wheels.

Forces.—It is assumed that the only force acting on the pendulum

* Eighty per cent. lead.

is the earth's gravitational attraction. It is unlikely that this is more variable than the earth's rotation. The direct attraction of the sun and moon are not large enough to produce sensible effects. The effect of tidal deformation on the earth's figure, and consequently on the earth's rotation and the gravitational attraction at any place have been investigated by Dr. Jeffreys in the *Geophysical Supplement*, vol. ii., p. 56, and found to produce as the largest term one similar to the semi-annual nutation, but only of one-tenth the amount, and therefore negligible. In countries where violent earthquakes take place, their effect on clocks must be considerable, but no such effects have been traced at Greenwich. It is possible that the irregularities of the order of $\pm$ s·01 shown in the comparisons of the two clocks may be of this origin.

It has been suggested that as invar is an alloy of nickel and steel it is magnetic, and that the rate of the pendulum may be influenced by the earth's magnetic field. Nickel-steel alloys have very little power of retaining permanent magnetisation, and the earth's magnetic field is too small for any permanent magnetisation to be produced. The effect of magnetic fields on the rates of chronometers and watches was carefully discussed, both from a practical and a theoretical point of view, by Chapman and Lewis in *M.N.*, **73**, 583, in which fields many times larger than the earth's field were used. They concluded that the only appreciable effect was in the temporary magnetisation of the steel bar of the balance wheel, and the analysis given is applicable to the pendulum rod. The formula (16) given by them shows that the effect of a field of strength F along the direction of equilibrium produces a change of daily rate of $42\mu T^2 \frac{A}{Ml} F^2 f(\theta_0)$ seconds, where μ is the permeability, T the period of a complete vibration, A the cross-section of the bar, M the mass of the pendulum, l its length, and $f(\theta_0)$ a function of the arc of vibration, which may be given the value 3·1 for very small arcs. The units are those of the C.G.S. system. If we put μ = 100, T = 2, A = 1, M = 7000, l = 100 we get a daily change of rate of s·072F². The value of μ used is that used by Chapman and Lewis for steel—I have no knowledge of its value for invar in weak magnetic fields, but its value is probably not higher. If we take F = ·43, the earth's vertical component, we get a rate of 0s·013 which should not vary from year to year. We therefore deduce that magnetic influences on the clock are negligible. In all cases the effect of magnetisation would be to accelerate the clock, while in 2 years the daily rate of the clock has been retarded by s·27 as the result of secular changes actually taking place.

Other Shortt Clocks.—Although several other Shortt clocks have been in use for a few years, so far as I know no detailed accounts of their performance have been published except by Professor Sampson, who has given details of Shortt 0 and Shortt 4 in the *Proceedings of the Royal Society of Edinburgh*. The difficulties regarding time determinations, to which he pays considerable attention, appear to have been largely overcome at Greenwich by reversing the transit on all stars and by the use of a striding level. The short-period irregularities in the com-

parison of the two clocks of the order of $\pm$ s·01 are very similar to those shown by the Greenwich clocks. The discovery of the cause of these irregularities should form an important stage in the obtaining of still more regular clocks, and observations such as contemplated by Mr. Loomis with Shortt clocks in America may help in the solution. Mr. Loomis proposes to make a direct comparison of the master pendulums to much smaller quantities than s·001.

For Shortt 0, Professor Sampson's data show no secular acceleration for the first half of 1927, and less than half that of the Greenwich clocks for the second half. For Shortt 4, on the other hand, the secular term appears to have a constant value 50 per cent. in excess of that of the Greenwich clocks.

The principal difference between the clocks so far disclosed is that for Shortt 4, Professor Sampson finds the same change of rate when the pressure was reduced by ·3 inches of mercury from 1·2 inches as from 1·5 inches. The change of rate was s·300 per day for 1 inch of mercury. With Shortt 3, the change was only at half this rate when the pressure was reduced from 1·4 to 1·0 inches, and only an eighth when reduced from 1 inch to 0·4 inches. The actual performance of Shortt 4 appears to be about the same as Shortt 3 and of Shortt 0 as of Shortt 11.

The clock Shortt 16 was set up as a mean time clock at Greenwich at the end of 1927. It is corrected several times daily as it controls the time signals sent out from the observatory. Also the case was opened in 1928 July for alteration of the rate. The true performance of this clock is therefore subject to large uncertainty, but it appears that its secular term is only about one-third that of Shortt 3 or 11, while its temperature coefficient is about twice as large.

The Earth's Rotation.—In addition to a secular slowing down of the earth's rate of rotation, a considerable mass of evidence has been collected which has been interpreted as indicating fluctuations in the rate of rotation. As the most rapidly moving body, the moon should show such irregularities most easily, and, as is well known, there are unexplained irregularities of comparatively large amount between the actual motion of the moon and the best lunar tables.

The results have been carefully discussed by Professor E. W. Brown in *Yale Transactions*, vol. iii., part 6, and by Professor de Sitter in *B.A.N.*, 124. The data obtained from the moon are fairly well substantiated by that from other bodies in the solar system, but it must be admitted that the quantities to be observed for the latter are so small as to approach the limit of accuracy of the observations themselves. If indeed the differences between the observed and computed positions of the moon are due to irregularity in the earth's rotation, that rotation must have been gaining about 1s·6 yearly round about 1870, and losing 0s·7 yearly in 1905, while changes as great as 1s yearly may have taken place quite suddenly.

The difficulty in testing such supposed changes in the rate of the earth's rotation by means of clocks is twofold—to get a clock which will (1) steadily maintain its rate within a fraction of a second a year, and (2) keep on doing this for a number of years. If indeed the rate of the

earth's rotation changed suddenly, as has been suggested, it would only be necessary to keep the clock going for two or three years at the critical epoch. If it should be possible to construct a clock to surmount (1) above but not (2), the difficulty in correlating its rate with that indicated by the moon's motion would be the difficulty of obtaining monthly positions of the moon correct to, say, $\pm$ 0″·05. The moon in fact is not suitable for checking irregularities if they should exist of the kind shown in the diagrams of this paper.

Although the residual irregularities of Shortt 3 and Shortt 11 do not agree, they appear to indicate that the rotation of the earth in 1928 did not differ by 1^s from that in 1927. Now the residuals between the observed and computed longitude of the moon for 1921, 1922, and 1923 * were

1921·5 . . .	— 9″·42
1922·5 . . .	— 9·17
1923·5 . . .	— 10·73.

Should such a change be observed between 1927·0, 1928·0, and 1929·0, it will not be possible to attribute it to irregularity in the earth's rotation, as 1″ in the moon's longitude corresponds to 2^s in the earth's rotation. These clocks should be able to decide whether erratic changes of 1″·0 in the moon's longitude of one year compared with the preceding and following years is due to the earth's rotation or not.

The Greenwich clocks have throughout been in the care of Mr. Bowyer.

The Solar Rotation derived from the H and K Lines in Prominences. (Second Paper.) By J. Evershed, F.R.S.

In *Monthly Notices*, **88**, 126, I summarised the results of measures of 92 spectra of prominences obtained in the years 1926–27. These yielded values of the solar rotation in four zones of heliographic latitude which are remarkable for the very large angular motion in each zone, as compared with values derived from spectra of the reversing layer, or from the motion of sunspots. In addition, the general shift towards red of the H and K lines was found to exceed the predicted relativity shift by about 0·003A, in approximate agreement with measures previously obtained with the same lines in the chromosphere, and with iron lines in the reversing layer in the same spectral region.

During 1928 the work has been continued on the same lines, but one change has been made in the method which greatly simplifies the process of measurement and reduction of the spectra. Owing to this change, and to the favourable season, a much larger number of plates

* It should be noted, however, that this is a rather erratic set of residuals, and the comparison of the observations with theory was direct in 1923 but indirect in 1921 and 1922. The data are taken from *Yale Transactions*, vol. iii., part 6.

has been dealt with, and the results now obtained, both as to the rotation and the general shift, are, I think, of considerable interest.

In the 1927 series the Fe arc spectrum was impressed on all the plates, and the positions of the prominence lines H and K were determined with reference to the terrestrial iron lines 3969·262 and 3930·300 respectively. This involved the measurement of two more Fe lines, in order to derive accurate factors for computing the positions of the prominence lines. For the estimate of the Einstein effect it was necessary to assume that the wave-lengths of terrestrial H and K, as determined by St. John, correctly represented the positions in air to the third decimal in angstroms.

In the new series of plates a comparison spectrum in which the calcium lines are prominent was employed, and the small shifts directly observed between the terrestrial and solar H and K lines were measured. This involves no subsidiary measures. The scale of each plate is given by the distance separating H and K, and the factors at H and at K are easily derived with sufficient accuracy from the mean factor as determined midway between H and K.

The substitution of the Ca^+ lines for Fe can have no appreciable effect on the values obtained of the solar rotation, except possibly a very small systematic error in the direction of increasing the displacements, due to the fact that the shifts are directly observed, and past experience has shown that in measuring small displacements there is an unconscious tendency to exaggerate them.* While every effort has been made to minimise this source of error, there remains the possibility that the values obtained are too large by an amount which would not exceed 5 per cent.

The 1927 results are entirely free from this source of error, because during the measures nothing is known or seen of the shifts, which are determined later by calculation.

The general shift towards red of the lines is not affected by this error, which does not increase in proportion to the amount of the shift, so that in taking the difference of shifts west − east it is eliminated. The direct observation of solar and terrestrial lines should yield a more satisfactory value of the Einstein effect than the indirect method involving assumed wave-lengths of the terrestrial H and K lines.

The carbon arc provides the source of Ca^+ radiation, but the character of the lines varies in carbon rods of different make, and it is necessary to select material that gives sharp narrow lines and as free as possible from iron lines. One particular set of rods 5 mm. in diameter was used throughout. The yellow flame of the arc from these yielded extremely narrow Ca lines, as well as the D lines of sodium and the aluminium lines.

The observed shifts converted into angstrom units are corrected for the annual and diurnal motions of the Earth in the direction of the Sun, computed for the date and time each spectrum was photographed. The heliographic latitudes of the prominences and the heights above the limb of the spectral images are determined as explained in my previous paper.

* *Kodaikanal Observatory Bulletins*, **3**, 23.

Rotation Results.—About 380 prominence spectra were photographed during the year, many of the plates having more than one measurable image impressed. Of these, 289 prominence images were measured, the others having ill-defined or greatly distorted lines due to violent motion in the line of sight. Those selected vary in quality, and I divide them into two classes, viz., 180 of class A, which have straight narrow lines, and 109 of class B with more or less ill-defined or wavy lines. In analysing the results these classes have been treated separately, with the result that the B spectra yield values of the rotation shift 15 per cent. larger than the A spectra, and the general shift nearly 14 per cent. larger.

I give first the result of a general average of all the east and west spectra of class A only, as in this series it so happens that the distribution in latitude is identical for the east and the west.

TABLE I.

Mean Shifts from all Measures of Class A Spectra.

	Mean Latitude.	K.	H.
West	21°·0	+0·0408A	+0·0409A
East	21°·1	−0·0152	−0·0164
Half sum		0·0280	0·0286
Mean of H and K			0·0283A=2·150 km./sec.
Mean correction for revolution of Earth . . .			+0·136 „
Mean correction for inclination of Sun's axis . .			+0·013 „
Sidereal rotation		=V	=2·299 km./sec.
Daily rotation at mean height 34″ . .		=ξ	=16°·9

Comparing this result with the value obtained from the 1927 series, a reduction of daily angular motion from 20° to 16°·9 is shown. There is still, however, a large excess over the corresponding motion for sun-spots, which for latitude 21° would be 14°. If the B spectra are included, 0·1 km./sec. is added to V, and ξ becomes 17°·6.

Considering that quantity rather than quality is important in measures of this kind, where large deviations from the mean values are found, I have included all the measures of class B spectra in the further analysis into zones of latitude, which are given in Table II.

TABLE II.

Solar Rotation in Different Latitudes.

No. of Measures.	ϕ.	h.	V.	ξ.
	°	″		°
99	9	41	2·48 km./sec.	17·1±0·8
51	18	28	2·36 „	17·1±0·9
78	25	33	2·66 „	20·2±0·7
37	35	32	1·98 „	16·6±1·7
24	51	35	1·59 „	17·3±2·4

The symbols used in this table have the following meanings :—

ϕ = mean latitude of each group of spectra.
h = mean height of each group of spectra.
V = mean velocity corrected to its sidereal value and for mean inclination of the Sun's axis.
ξ = daily angular motion in degrees.

The formula used for computing ξ takes account of the mean height in each zone, whereby the Sun's radius is increased 3 per cent. or more. The probable errors are derived from the residuals of the individual measures in each group, taking the mean values given by H and K. These probable errors are rather arge considering the large number of measures, and indicate the large departures from the mean values due to the random motions of the prominences, and not to the measuring errors, which are relatively small.

These values of ξ are in surprisingly good agreement with those obtained from the 1927 series. They do not indicate any diminution of angular speed, except in the 9° zone. In other zones the figures are much the same for the two years.

A curious feature is the large value of ξ for the 25° zone, showing strongly in both years, and this region in 1928 gave more consistent values than any other zone, as is indicated by the probable error. There were 53 spectra of class A and 25 of class B in this zone, but each class gives the same mean value. If the north and south hemispheres are treated separately, it is found that this high value of the angular speed occurs in the southern 25° zone but not in the northern, where the maximum value of ξ is found in the 18° zone. It would be interesting to learn whether these features have any relation to sunspot frequency, or are purely accidental.

There is no evidence in this series of a polar retardation of angular motion, and excepting for the large value in the region of 25° the figures indicate a remarkable constancy.

The prominence spectra have been photographed at various heights above the chromosphere, ranging from 5″ to 150″, and from the large number of measures now available the effect of height on angular motion may be estimated. Dividing the prominence spectra of each zone into two series, in which the mean heights are about 20″ and 60″, I find the values obtained are extremely irregular in the different zones, some giving larger velocities for the higher prominences, and others the reverse. Yet on the whole, counting the two years together, there is a mean excess for the high prominences, amounting to about +0·17 km./sec., or an increase of 7 per cent. in the mean observed velocity. The increase of height from 20″ to 60″ demands a 4 per cent. increase in velocity for the same angular motion. The evidence therefore favours an increase of angular motion with height, but I do not consider that much weight can be given to this result.

General Shift of the H and K Lines in Prominences.—The mean shift

of the lines towards red in the class A series may be taken from the differences west—east in Table I. These give :—

$\frac{W-E}{2}$ =	K.	H.
	+0·0128	+0·0122

Mean	+0·0125
Add for pressure effect .	+0·0017
Sun—arc in vacuum . .	+0·0142A±·0007

The results for the different groups of latitude, in which spectra of class B are included, are shown in Table III.

TABLE III.

ϕ.	Mean of H and K.	No. of Measures.
9°·2	+0·0137±·0016	99
18	+0·0140±·0017	51
25	+0·0127±·0013	78
35	+0·0120±·0027	37
51	+0·0190±·0029	24
Weighted mean	+0·0137±·0009	
Add for pressure effect . . .	+0·0017	
Sun—arc in vacuum . . .	+0·0154A	

Omitting the discrepant value at latitude 51° the mean is

	+0·0132±·0003
Add for pressure effect	+0·0017
Sun—arc in vacuum	+0·0149A

The most reliable result is perhaps that given by the A spectra alone, which is, omitting the last figure, +0·014A. My previous result from the 1927 spectra was +0·011A. I am unable to account for this large difference. It would seem to imply that the H and K arc lines in my spectra were displaced to violet by 0·003A, or that the assumed wave-lengths of these lines are in error by this amount, the larger shift being nearer the truth.

I had rather confidently expected that with the much larger amount of material dealt with the shift would be brought into better agreement with the Einstein effect, that is to say, it would be reduced by something of the order of 0·003A. Instead, it is increased by this amount, making the discrepancy 0·006A. A redetermination of the wave-lengths of H and K, as produced in the carbon arc and with very high dispersion, is now on my programme, and an attempt will be made to redetermine the shift given by the H and K lines in the chromosphere.

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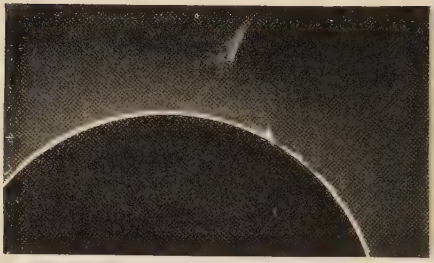

Fig. 1. $7^h\ 52^m$.

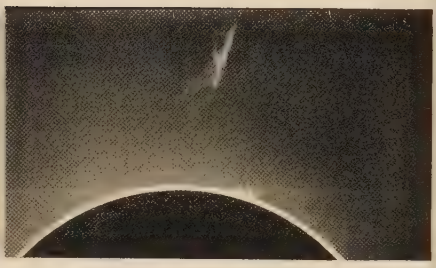

Fig. 2. $8^h\ 35^m$.

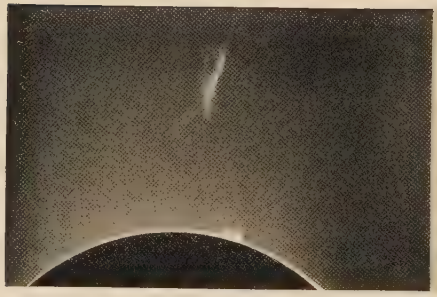

Fig. 3. $8^h\ 45^m$.

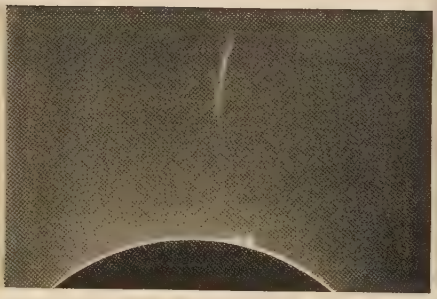

Fig. 4. $8^h\ 52^m$

Fig. 5. $8^h\ 58^m$.

Fig. 6. $9^h\ 3^m$.

Prominence of 1928 November 19. Kodaikanal Observatory.

A Very High Prominence on 1928 *November* 19.

By T. Royds, D.Sc.

(Plate 6.)

An eruptive prominence photographed on 1928 November 19 breaks all records in the experience of Kodaikanal as regards not only the height attained (567,000 miles) but also the brightness at great heights above the chromosphere. Prominences of the order of half a million miles in height above the surface of the sun have occasionally been recorded previously, notably the prominence of 1916 May 26 photographed simultaneously at Kashmir and Kodaikanal,* but in previous experience the prominence was rapidly dissipating itself at these great heights, whereas on 1928 November 19 the prominence remained compact and bright until the time when the clouds intervened.

Observing conditions were never very good on that day since at the best the sun shone through cirrus clouds. Some interruptions were caused by light cumulus clouds passing in front of the sun; the last photograph (at $9^h\ 12^m$ I.S.T.) was taken through continuously passing cloud, and immediately afterwards thick cumulus clouds completely covered the sky. Whilst regretting that the final phases of this prominence were lost when its ultimate height doubtless greatly exceeded all previous records, one can be grateful for the timely break in the monsoon which revealed it. Six photographs reproduced in figures 1–6 are selected to illustrate this prominence. It was already well developed when the first photograph was obtained at $7^h\ 52^m$ (fig. 1), reaching to the limit of the field of view of the spectroheliograph at a height of 8′·5 above the surface of the sun. Its base was at 71°–72° in the S.W. quadrant; the small prominence at 46°–52° remained practically unchanged throughout. The presence of the prominence was not suspected until the first plate had been developed, but from $8^h\ 26^m$ the prominence was continuously photographed until clouds stopped operations. The times given (Indian Standard Time $5\frac{1}{2}$ hours ahead of Greenwich) are the approximate times when the slit of the spectroheliograph reached the limb of the sun, but the exact times when the slit passed each portion of the prominence were used for determining the motions of different parts of the prominence. In fig. 6 ($9^h\ 3^m$) the maximum height reached was 18′·3 or 495,000 miles; in the photograph taken at $9^h\ 12^m$ (which is not suitable for reproduction on account of clouds) the maximum height was 20′·9 or 567,000 miles. Generally speaking, the velocities were smallest in the lower parts of the prominence (60–70 kms./sec.) and greatest in the higher parts (100–170 kms./sec.), and the velocities increased with time; the greatest velocity amounted to 229 kms./sec. at the top of the prominence between the last two photographs. Although the motion was largely one of ascent, the movements of all parts appeared to be directed away from points on the chromosphere within 2° of − 67° W.

* J. Evershed, *Kodaikanal Observatory Bulletin*, No. 55.

Owing to bad weather conditions on the days preceding it was not possible to trace whether there was a connection between the eruption at the limb and any features previously seen on the disc.

Kodaikanal:
1928 *December* 3.

Greenwich Observations of the Sun and the Four Outer Planets from 1901–1928. By Sir F. W. Dyson and R. T. Cullen.

As the residuals have indicated peculiar variations from year to year, it has been thought desirable to reduce the observations as far as possible to a uniform system.

For the R.A.'s Newcomb's system has been used at Greenwich since 1903. A correction of $+ {}^{s}\cdot054$ has been applied to the Greenwich R.A.s for 1901–02 to reduce them to this system.

It should be noted that there was a discontinuity in the method of observing transits at the beginning of 1915, when the self-registering micrometer was introduced.

The declinations have all been reduced to the system of Pulkova refractions, colatitude 38° 31′ 21″·90, and flexure $+ 0''\cdot60 \sin z$ south.

The corrections for latitude variations have been applied: for 1901–14 from the International results, and since then from observations at Greenwich with the Cookson zenith telescope.

I. Sun.

The residuals (Observed *minus* Tabular) in R.A. and Dec. have been collected for each month and the means taken for series of years, viz.

1901–05
1906–14
1915–21
1922–27

The dividing dates are those when observations for a new Greenwich Catalogue were commenced.

In forming the equations of condition the R.A. of the Sun was taken as 295° for January, and 30° added to this for each successive month. The declinations were taken to correspond with these right ascensions.

The equations of condition are given in Tables I. and II., where γ is the constant correction to the assumed R.A.'s of the Clock Stars to reduce them to the Sun's node as equinox, that is, the correction to Newcomb's system of R.A.'s; $d\odot$ the correction to the tabular mean longitude of the Sun; $d\omega$ the correction to the obliquity; dh and dk corrections to $h = e \cos \varpi$; and $k = e \sin \varpi$, where e is the eccentricity and ϖ the longitude of perigee.

In Table II. $\Delta\delta_0$ is the correction to the equator point of the Transit Circle.

In forming these tables, the weights for different months ranged from 2 to 4 according to the number of observations.

TABLE I.

R.A. (*Newcomb's Equinox*).

	γ	$d\odot$.	$d\omega$.	dh.	dk.	Unit s·001 1901 to 1905.	1906 to 1914.	1915 to 1921	1922 to 1927.
Jan.	−1	+1·06	+·17	−1·95	− ·83	=− 6	+52	+113	+115
Feb.	−1	+ ·97	+·20	−1·18	−1·55	=+16	+55	+101	+100
Mar.	−1	+ ·92	+·04	− ·17	−1·83	=+ 5	+41	+ 89	+103
Apr.	−1	+ ·95	−·17	+ ·86	−1·69	=+ 5	+48	+ 82	+110
May	−1	+1·04	−·20	+1·74	−1·12	=−14	+36	+ 93	+111
June	−1	+1·09	−·04	+2·17	− ·17	=−23	+43	+ 96	+111
July	−1	+1·06	+·17	+1·95	+ ·83	=+10	+44	+125	+128
Aug.	−1	+ ·97	+·20	+1·18	+1·55	=+12	+59	+128	+126
Sept.	−1	+ ·92	+·04	+ ·17	+1·83	=+ 8	+74	+114	+127
Oct.	−1	+ ·95	−·17	− ·86	+1·69	=+30	+52	+120	+131
Nov.	−1	+1·04	−·20	−1·74	+1·12	=+ 5	+78	+113	+133
Dec.	−1	+1·09	−·04	−2·17	+ ·17	=+10	+47	+120	+124

TABLE II.

	$\Delta\delta_0$.	$d\odot$.	$d\omega$.	dh.	dk.	Dec. 1901 to 1905.	1906 to 1914.	1915 to 1921.	1922 to 1927.
						″	″	″	″
Jan.	1	+·17	− ·91	−·31	−·13	=−·07	+·27	+·22	+·15
Feb.	1	+·33	− ·57	−·39	−·52	=−·22	+·19	+·23	+·33
Mar.	1	+·40	− ·09	−·08	−·79	=−·24	−·02	+·10	+·22
Apr.	1	+·36	+ ·42	+·33	−·64	=−·13	+·05	+·03	+·04
May	1	+·23	+ ·82	+·39	−·25	=+·25	+·26	+·14	·00
June	1	+·03	+1·00	+·07	−·01	=−·05	−·25	−·05	−·21
July	1	−·17	+ ·91	−·31	−·13	=+·23	+·13	−·13	−·13
Aug.	1	−·33	+ ·57	−·39	−·52	=+·18	+·17	−·05	−·28
Sept.	1	−·40	+ ·09	−·08	−·79	=−·21	−·01	−·28	−·50
Oct.	1	−·36	− ·42	+·33	−·64	=−·09	−·11	−·30	−·87
Nov.	1	−·23	− ·82	+·39	−·25	=+·05	−·12	−·16	−·23
Dec.	1	−·03	−1·00	+·07	−·01	=+·12	+·17	+·10	−·05

The normal equations obtained from Tables I. and II., giving equal weight to each month, are :

From R.A.'s.

							1901 to 1905.	1906 to 1914.	1915 to 1921.	1922 to 1927.
12	γ	$-12{\cdot}06d\odot$	$\cdot00d\omega$	$\cdot00dh$	$\cdot00dk$		$=-''{\cdot}87$	$-9''{\cdot}43$	$-19''{\cdot}41$	$-21''{\cdot}28$
$-12{\cdot}06$		$+12{\cdot}16$	$\cdot00$	$\cdot00$	$\cdot00$		$=+\cdot81$	$+9{\cdot}43$	$+19{\cdot}56$	$+21{\cdot}39$
$\cdot00$		$\cdot00$	$+\cdot28$	$\cdot00$	$\cdot00$		$=+\cdot04$	$\cdot00$	$+\cdot15$	$-\cdot04$
$\cdot00$		$\cdot00$	$\cdot00$	$-27{\cdot}40$	$\cdot00$		$=-1{\cdot}50$	$-1{\cdot}35$	$-\cdot89$	$-\cdot35$
$\cdot00$		$\cdot00$	$\cdot00$	$\cdot00$	$+21{\cdot}16$		$=+1{\cdot}26$	$+1{\cdot}72$	$+2{\cdot}82$	$+2{\cdot}36$

From Dec.'s.

12	$\Delta\delta_0$	$\cdot00d\odot$	$\cdot00d\upsilon$	$\cdot00dh$	$-4{\cdot}68dk$	$=-''{\cdot}18$	$+''{\cdot}73$	$-''{\cdot}15$	$-1''{\cdot}53$
$\cdot00$		$+\cdot96$	$\cdot00$	$\cdot00$	$\cdot00$	$=-\cdot17$	$+\cdot15$	$+\cdot50$	$+\cdot90$
$\cdot00$		$\cdot00$	$+6{\cdot}02$	$\cdot00$	$\cdot00$	$=+\cdot48$	$-\cdot18$	$-\cdot29$	$-\cdot26$
$\cdot00$		$\cdot00$	$\cdot00$	$+1{\cdot}04$	$\cdot00$	$=+\cdot07$	$-\cdot24$	$-\cdot13$	$-\cdot39$
$-4{\cdot}68$		$\cdot00$	$\cdot00$	$\cdot00$	$+2{\cdot}76$	$=+\cdot43$	$-\cdot21$	$+\cdot21$	$+\cdot80$

Combined R.A.'s and Dec.'s.

								1901–05.	1906–14.	1915–21.	1922–27.
12	γ	..	$-12{\cdot}06d\odot$	$\cdot00d\omega$	$\cdot00dh$	$\cdot00dk$		$=-''{\cdot}87$	$-9''{\cdot}43$	$-19''{\cdot}41$	$-21''{\cdot}28$
..	12	$\Delta\delta_0$	$\cdot00$	$\cdot00$	$\cdot00$	$-4{\cdot}68$		$=-\cdot18$	$+\cdot73$	$-\cdot15$	$-1{\cdot}53$
$-12{\cdot}06$		$\cdot00$	$+13{\cdot}12$	$\cdot00$	$\cdot00$	$\cdot00$		$=+\cdot64$	$+9{\cdot}58$	$+20{\cdot}06$	$+22{\cdot}29$
$\cdot00$		$\cdot00$	$\cdot00$	$+6{\cdot}30$	$\cdot00$	$\cdot00$		$=+\cdot52$	$-\cdot18$	$-\cdot14$	$-\cdot30$
$\cdot00$		$\cdot00$	$\cdot00$	$\cdot00$	$+28{\cdot}44$	$\cdot00$		$=-1{\cdot}43$	$-1{\cdot}59$	$-1{\cdot}07$	$-\cdot74$
$\cdot00$		$-4{\cdot}68$	$\cdot00$	$\cdot00$	$\cdot00$	$+23{\cdot}92$		$=+1{\cdot}69$	$+1{\cdot}51$	$+3{\cdot}03$	$+3{\cdot}16$

The solution of the combined equations is :

	1901–05. s	1906–14. s	1915–21. s	1922–27. s
γ	$-\cdot020$	$-\cdot046$	$-\cdot070$	$-\cdot057$
$\Delta\delta_0$	$+''{\cdot}01$	$+''{\cdot}09$	$+''{\cdot}04$	$-''{\cdot}08$
$d\odot$	$-\cdot23$	$+\cdot10$	$+\cdot56$	$+\cdot91$
$d\omega$	$+\cdot08$	$-\cdot03$	$-\cdot02$	$-\cdot05$
dh	$-\cdot05$	$-\cdot06$	$-\cdot04$	$-\cdot03$
dk	$+\cdot07$	$+\cdot08$	$+\cdot14$	$+\cdot12$

From dh and dk we find

de	$-''{\cdot}08$	$-''{\cdot}09$	$-''{\cdot}14$	$-''{\cdot}12$
$ed\varpi$	$-\cdot04$	$-\cdot04$	$-\cdot01$	$\cdot00$

The values of the quantities $\Delta\lambda = d\odot$ and $\Delta'\alpha = -\gamma + d\odot$ for the separate years will be found in a paper by Dr. Jackson and Mr. Cullen (*M.N.*, **87**, 459). The most important change appears to be in $d\odot$. This indicates that in about twenty-five years the mean longitude of the Sun has increased 1″·3 more than in Newcomb's tables. The regularity of this increase in $d\odot$ is noticeable.

Dr. Cowell was consulted as to whether an error in the tables was possible for the twentieth century in reference to the nineteenth. This he negatived but suggested that the secular acceleration of the Sun was shown by the observations. Accordingly comparison was made with the results from 1750 onwards as given by Newcomb.

Comparisons of the observed longitudes of the Sun with Newcomb's tables have been made by Dr. Ross and Dr. Fotheringham. Both of them have combined observations of right ascension with observations of declination. The advantage of working with declinations alone is that the personalities in observations of the Sun's right ascension are irregular.

The corrections to the Sun's longitude as given by Leverrier's tables from 1753–1892 are given by Newcomb (*Astronomical Constants*, p. 30). To these have been added those for later years, uncorrected, as are Newcomb's figures, for variation of latitude.

Years.	*dl*.	O—C.	wt.	Years.	*dl*.	O—C.	wt.
	″	″			″	″	
1753–57	+0·78	− ·28	1	1847–52	+0·25	+ ·42	4
58–62	+1·50	+ ·34	1	53–58	+0·55	+ ·75	5
65–70	−0·23	−1·05	1	59–64	+0·03	+ ·26	5
71–78	+0·48	− ·21	1	65–70	−0·23	+ ·01	5
79–85	+1·23	+ ·66	1	71–76	−0·15	+ ·11	5
86–91	+0·48	+ ·02	1	77–82	−0·90	− ·63	5
92–97	+1·12	+ ·74	1	83–88	−0·27	·00	5
98–1803	+0·41	+ ·11	1	89–92	−0·05	+ ·22	5
1804–10	+0·18	− ·03	1	93–96	−0·78	− ·52	3
12–16	−0·15	− ·28	3	97–1900	−0·76	− ·51	3
17–22	−0·41	− ·48	3	1901–05	−0·79	− ·55	4
23–28	+0·43	+ ·42	3	06–14	−0·51	− ·28	5
29–34	−0·08	− ·04	3	15–21	+0·20	+ ·39	5
35–40	−0·12	− ·03	3	22–27	+0·55	+ ·70	5
41–46	+0·21	+ ·34	3				

Equating the values of *dl* to the formula

$$a + b\mathrm{T} + c\mathrm{T}^2,$$

where T is measured in centuries from 1850, we find

$$dl = -\,''\cdot17 - ''\cdot56\mathrm{T} + ''\cdot78\mathrm{T}^2.$$

The probable error of the coefficient of T^2, calculated from the squares of the residuals, is $\pm\,0''\cdot14$.

These figures are incorporated in the diagram. The formula is represented by the curved line. The horizontal broken line gives Leverrier's base, and the inclined broken line is Newcomb's correction to Leverrier.

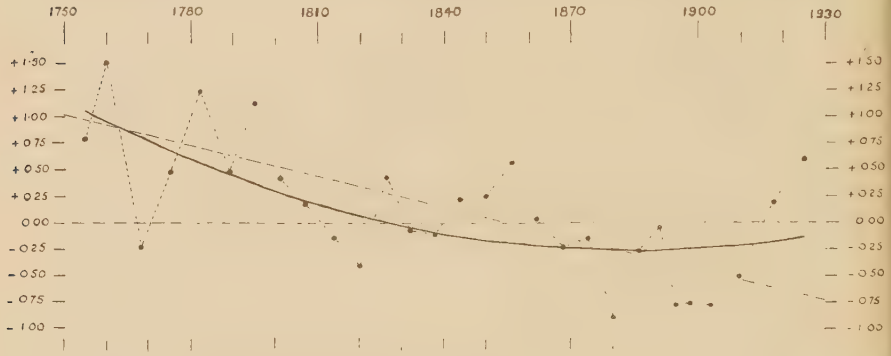

FIG. 1.—Sun's mean longitude from declination observations 1750–1927. Observed *minus* computed.

The residuals between the formula and the observations are of a somewhat systematic character. If a smooth curve be drawn through the plotted points, maxima and minima differences between this curve and the heavy one representing the formula are as given below. Similar quantities taken from the observed fluctuations of the Moon as given in *M.N.*, **83**, 364 (supplemented by data for the last few years) are included for comparison. It will be seen that the maximum departures for Sun and Moon are approximately at the same epochs and about 1 to 10.

Observed – Computed.

Epoch (approx.)	Sun.	Moon.	Epoch (approx.).
	″	″	
1790	+·35	+2·6	1790
1820	−·20	−2·3	1820
1855	+·45	+4·0	1860
1897	−·45	−4·6	1895
1925	+·65	+3·9	1920

II. The Four Outer Planets.

The results given are geocentric longitudes and latitudes derived from the observed R.A.'s and Dec.'s reduced to a uniform system similar to that adopted for the Sun.

The values depend upon the mean difference between the observed and computed positions in R.A. and Dec. for each apparition. The mean time of observation is about two hours before midnight, or, roughly, 30° after opposition.

The positions tabulated in the *Nautical Almanac* are based on :
Jupiter } Hill's Tables (*Ast. Papers of the American Eph.*, vol. vii.,
Saturn } pts. 1–2).
Uranus } Newcomb's Tables (*Ast. Papers of the American Eph.*,
Neptune } vol. vii., pts. 3–4).

Those for Jupiter and Saturn were introduced into the *N.A.* in 1901, for Uranus and Neptune in 1904.

The observed corrections to the tabular geocentric longitude and latitude in the sense "Observed *minus* Computed" since 1901 are :

Jupiter.			Saturn.			Uranus.			Neptune.		
Date.	Long.	Lat.	Date.	Long.	Lat.	Date.	Long.	Lat.	Date.	Long.	Lat.
1900+	″	″	1900+	″	″	1900+	″	″	1900+	″	″
1·6	+ ·1	+·2	1·6	+ ·1	+ ·1						
2·7	+ ·7	+·1	2·6	− ·2	+ ·1						
3·8	+ ·4	−·5	3·7	− ·3	+ ·3						
4·9	+ ·6	−·6	4·7	+ ·6	+ ·2	4·5	+2·0	−·1	4·1	·0	− ·3
5·9	+ ·7	·0	5·8	+ ·4	·0	5·6	+1·1	−·7	5·0	+ ·7	·0
7·1	+ ·3	−·4	6·7	+ ·5	+ ·4	6·6	+1·8	−·1	6·0	+ ·1	− ·7
8·2	+ ·4	−·1	7·8	+ ·9	+ ·3	7·6	+2·0	−·4	7·1	−1·0	− ·4
9·2	+ ·4	−·2	8·8	+1·1	− ·3	8·6	+1·1	−·4	8·1	− ·8	− ·4
10·3	− ·2	−·1	9·9	+1·1	·0	9·6	+1·7	−·1	9·1	− ·7	− ·5
11·3	+ ·2	+·7	10·9	+ ·8	− ·1	10·6	+2·8	+·6	10·1	−1·0	− ·3
12·4	+1·0	+·4	11·9	+1·7	− ·4	11·6	+2·7	+·2	11·1	−1·3	+ ·2
13·6	+ ·7	+·2	12·9	+ ·6	− ·1	12·6	+1·8	+·4	12·1	−1·0	− ·2
14·7	+ ·5	+·4	14·0	+ ·4	− ·4	13·7	+3·1	−·2	13·1	−1·4	− ·4
15·8	+ ·5	−·1	15·0	+1·2	− ·3	14·7	+2·4	+·2	14·2	−1·0	− ·5
16·9	+ ·3	·0	16·0	− ·7	−1·0	15·7	+2·0	+·2	14·9	−1·6	− ·8
18·0	+ ·2	−·6	17·1	− ·8	− ·9	16·7	+2·0	·0	16·2	−2·0	− ·9
19·1	+ ·7	−·4	18·2	−1·4	− ·5	17·8	+1·8	+·4	17·2	−1·8	−1·0
20·2	+ ·8	−·6	19·3	−1·3	− ·7	18·8	+1·6	−·1	18·2	−2·1	−1·2
21·3	+ ·6	−·6	20·3	−2·0	− ·1	19·8	+1·9	+·4	19·2	−2·1	− ·8
22·4	+ ·2	−·4	21·4	−1·3	− ·5	20·8	+1·7	+·6	20·2	−2·1	− ·8
23·4	+ ·1	−·2	22·4	−1·9	− ·4	21·8	+1·9	+·3	21·2	−2·4	− ·9
24·5	− ·4	·0	23·4	−1·9	− ·6	22·8	+2·0	+·3	22·2	−1·7	− ·5
25·6	− ·8	−·1	24·4	−1·9	− ·5	23·8	+1·2	−·3	23·2	−2·3	− ·9
26·7	− ·9	+·3	25·4	−1·9	− ·1	24·9	+1·5	−·2	24·2	−2·7	−1·2
27·8	−1·4	−·2	26·5	−2·2	− ·6	25·9	+1·9	−·3	25·2	−2·7	− ·7
			27·5	−1·8	·0	26·9	+1·9	+·1	26·2	−2·8	− ·2
			28·5	−1·4	+ ·1	27·9	+1·4	+·4	27·2	−2·6	−1·2
									28·2	−3·4	− ·5

Each determination, with few exceptions, depends upon twelve or more observations.

The residuals are plotted in the following diagrams to illustrate clearly the observed deviations.

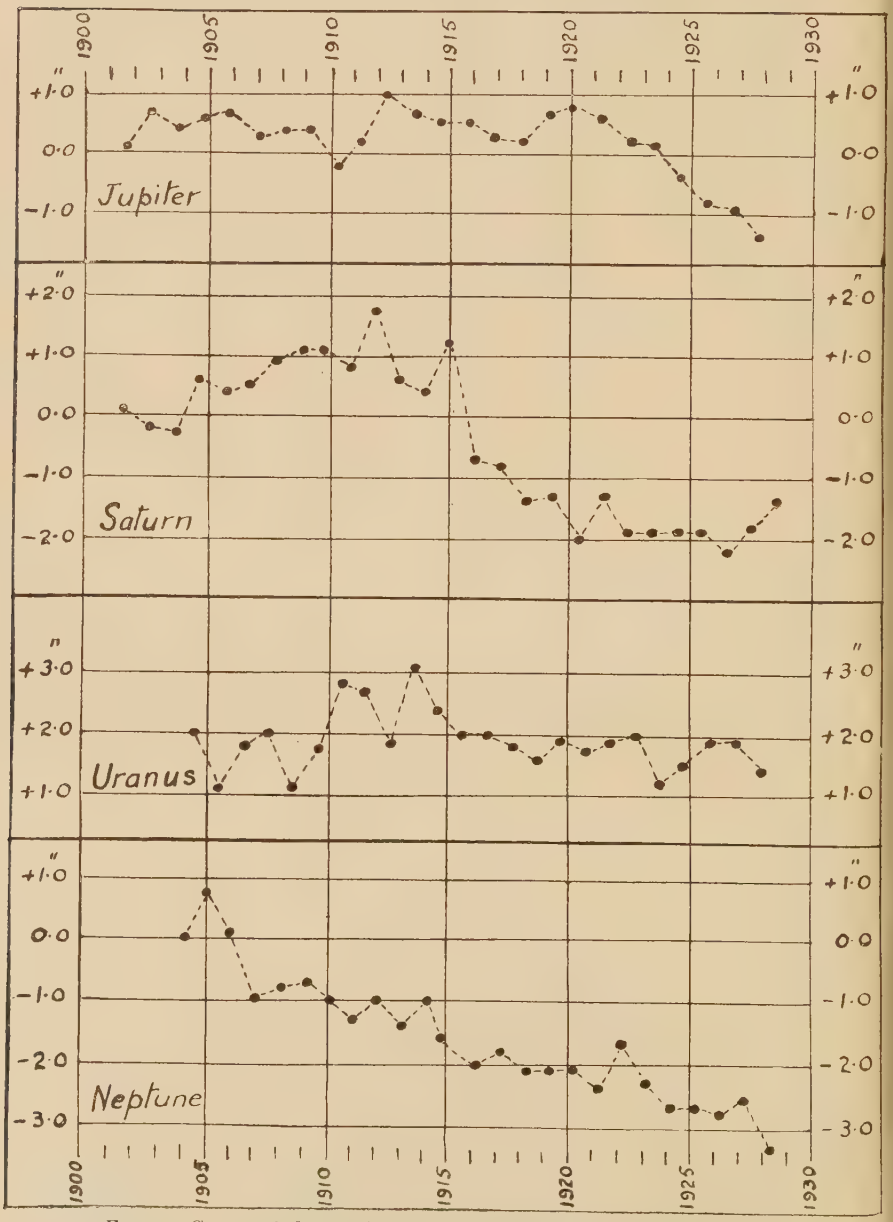

FIG. 2.—Geocentric longitude. Jupiter, Saturn, Uranus, and Neptune. Observed *minus* Computed.

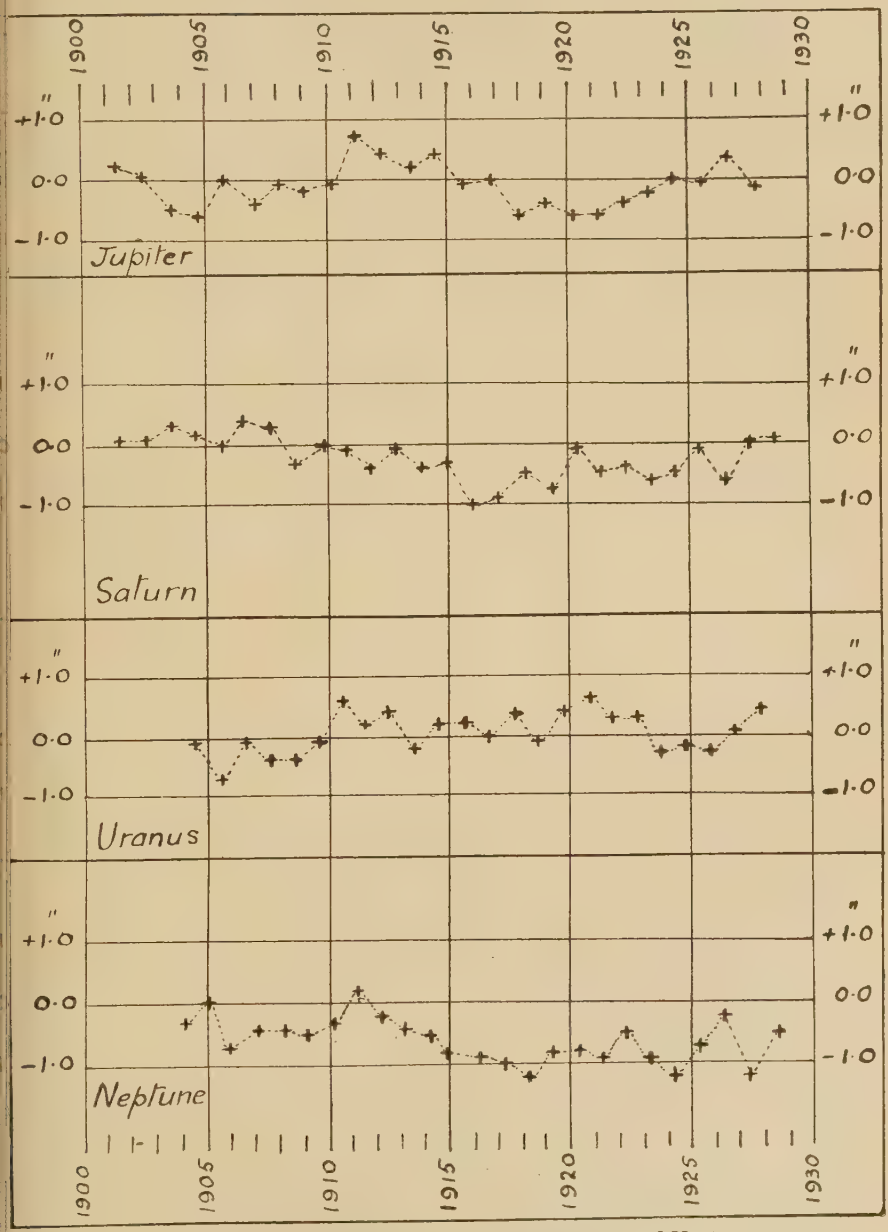

FIG. 3.—Geocentric latitude. Jupiter, Saturn, Uranus, and Neptune. Observed *minus* Computed.

The more important changes in the longitudes of the different planets are as follows :—

Jupiter.—The observed places were in advance of the predicted by ″·5 from 1901 to 1921. Since then the planet has fallen steadily behind, and now is 1″ behind.

Saturn.—Apparently there is a periodic variation of amplitude 1″·5 in thirty years, equal to that of the planet's revolution round the Sun.

Uranus.—A constant correction of about 2″ appears necessary to the tabular longitude.

Neptune.—The observations agreed with tabular places in 1904 when the tables were introduced. Since then the motion appears to have decreased, and now the planet is 3″ behind the tabular position. The latitude of Neptune requires a correction of approximately − 0″·7.

Note should be made of a change in observing transits in 1915. Its effects are not apparent in the longitudes except perhaps for Saturn. Whether in this case the rings may have some bearing or not on the observation is problematical.

Again in Saturn a small periodic term is noticeable. This might arise from an error in right ascension depending on declination, but there is nothing of this magnitude in the observed right ascensions of stars.

Mean Areas and Heliographic Latitudes of Sunspots in the Year 1927.

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those in the *Monthly Notices*, **88**, 83, and are derived from the measurement of photographs taken at the Royal Observatories of Greenwich and the Cape, and in India at the Kodaikanal Observatory. A photograph for July 9 was kindly lent for measurement by the Mt. Wilson Observatory.

Table I. gives the mean daily area of umbræ, whole spots, and faculæ for each synodic rotation of the Sun included in the year 1927, whilst the mean values for each year as a whole from 1923 to 1927 are given in Table II.

Table III. gives for each rotation in the year 1927 the mean daily area of the whole spots (corrected for foreshortening) and the mean heliographic latitude of the spotted areas for both hemispheres, northern and southern. The mean heliographic latitude of the entire spotted area and the mean distance from the equator of all spots are also tabulated. The mean values for the years 1923 to 1927 are given in Table IV. Tables II. and IV. are in continuation of similar tables given in the *Monthly Notices*; for the years 1874 to 1888, **49**, 381 and 382; 1889 to 1902, **63**, 465 and 466; 1901 to 1914, **76**, 402 and 403; 1913 to 1924, **85**, 1007 and 1008.

The rotations in Tables I. and III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node of the Sun's equator on the ecliptic at mean noon on 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days.

TABLE I.

No. of Rotation.	Rotation commenced G.C.T.	No. of Days on which Sun photographed.	Mean of Daily Areas. Projected.* Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening.† Umbræ.	Whole Spots.	Faculæ.
	1926.							
980	Dec. 20·41	28	370	2259	2337	285	1700	2807
	1927.							
981	Jan. 16·75	27	441	2787	2508	311	1973	3046
982	Feb. 13·09	27	165	844	2446	126	631	2916
983	Mar. 12·42	28	263	1499	2074	218	1244	2508
984	Apr. 8·72	27	310	1647	2285	218	1165	2757
985	May 5·97	27	263	1320	2004	197	993	2468
986	June 2·18	27	197	1185	1763	142	848	2119
987	June 29·38	28	221	1183	1354	165	882	1702
988	July 26·59	27	201	1039	1373	158	845	1729
989	Aug. 22·82	27	323	1921	1434	240	1387	1788
990	Sept. 19·07	27	204	1160	1588	167	941	1976
991	Oct. 16·36	28	153	889	1422	118	668	1738
992	Nov. 12·66	27	213	1267	1607	154	921	1939
993	Dec. 9·97	27	203	1095	1367	146	801	1665

TABLE II.

Year.	No of Days Photographed.	No of Days Without Spots.	Mean Daily Areas. Projected.* Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening.† Umbræ.	Whole Spots.	Faculæ.
1923	365	171	15	74	189	10	55	222
1924	366	97	62	361	498	46	276	575
1925	365	19	189	1102	1508	141	830	1750
1926	365	2	284	1651	2154	217	1262	2551
1927	365	0	248	1417	1810	186	1058	2212

* Expressed in millionths of the Sun's disc.
† Expressed in millionths of the Sun's hemisphere.

TABLE III.

No. of Rotation.	Rotation commenced G.C.T.	Spots NORTH of the Equator. Mean Daily Area.	Spots NORTH of the Equator. Mean Heliographic Latitude.	Spots SOUTH of the Equator. Mean Daily Area.	Spots SOUTH of the Equator. Mean Heliographic Latitude.	Mean Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
	1926.		°		°	°	°
980	Dec. 20·41	725	17·32	975	13·93	− 0·61	15·38
	1927.						
981	Jan. 16·75	789	20·31	1185	16·17	− 1·60	17·83
982	Feb. 13·09	189	14·47	441	15·40	− 6·42	15·12
983	Mar. 12·42	864	22·67	380	15·11	+11·12	20·36
984	Apr. 8·72	396	15·69	769	15·34	− 4·80	15·46
985	May 5·97	414	16·91	579	14·52	− 1·42	15·52
986	June 2·18	567	16·88	282	11·59	+ 7·42	15·12
987	June 29·38	425	16·75	457	8·97	+ 3·42	12·72
988	July 26·59	90	16·36	755	11·04	− 8·12	11·61
989	Aug. 22·82	141	15·82	1246	14·17	−11·12	14·34
990	Sept. 19·07	283	19·09	657	14·60	− 4·45	15·96
991	Oct. 16·36	152	16·93	515	10·48	− 4·23	11·95
992	Nov. 12·66	191	10·99	730	12·17	− 7·37	11·93
993	Dec. 9·97	222	10·66	579	11·85	− 5·62	11·52

TABLE IV.

Year.	No. of Days on which Sun photographed.	Spots NORTH of the Equator. Mean Daily Area.	Spots NORTH of the Equator. Mean Heliographic Latitude.	Spots SOUTH of the Equator. Mean Daily Area.	Spots SOUTH of the Equator. Mean Heliographic Latitude.	Mean Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
			°		°	°	°
1923	365	33	13·88	22	17·32	+ 1·34	15·26
1924	366	232	22·19	45	25·54	+14·49	22·73
1925	365	517	20·70	313	19·37	+ 5·60	20·20
1926	365	664	19·26	599	17·98	+ 1·60	18·65
1927	365	379	17·75	679	13·54	− 2·33	15·05

The principal features of the record for the year 1927 were as follows :—

1. A decrease, amounting to about 15 per cent., in the mean daily areas for spots and faculæ as compared with the mean areas for 1926.

2. The number of groups, on the contrary, showed an increase.

3. The southern hemisphere was markedly the more active.

4. The mean latitude of spots was 15° as compared with 18½° for the previous year ; latitudes ranged from 2° to 33°. Large spots were, on the average, about 5° nearer the equator in the southern hemisphere than in the northern.

5. On no days were spots and faculæ absent from the Sun's disc.

6. The number of separate groups lasting for at least two days was 343, 141 being in the northern hemisphere and 202 in the southern. Besides these, 105 very small groups (mostly consisting of one or two tiny spots) were measured on one day only. Fifty of these were in the northern hemisphere and 55 in the southern.

7. The year 1927 was one of considerable solar activity, but until the measures for 1928 are completed, the time of the maximum of the cycle is somewhat uncertain. It would appear that the maximum is a flat one, being spread over the three years 1926–28, with a double peak for the area curve in 1926 and 1928.

Royal Observatory, Greenwich :
1928 *December* 27.

Faint Stars in the Selected Areas with Large Proper Motion. Fourth List from the Radcliffe Observatory, Oxford. By H. Knox-Shaw, M.A., D.Sc., Radcliffe Observer.

This list is uniform with the three earlier ones published in *M.N.*, **86**, 342 ; **87**, 225 ; and **88**, 181, to the first of which reference should be made for explanation. The present list contains twenty-four stars, with proper motions as large as $0''\cdot1$ per annum from fifteen areas.

The proper motions given in these lists are subject to corrections for magnitude equation,* the application of which would bring into better accord the values for the two stars in Area 62 with a common proper motion.

The plates were measured and reduced under the superintendence of Mr. H. G. Scott Barrett.

Area No.	Area measured.	No. of Stars measured.	No. of Comparison Stars.	Mean Mag. of Comparison Stars.	Limiting Magnitude.	Star No	Magnitude.	R.A. 1900·0.	Dec. 1900·0.	Annual Proper Motion. μ_α.	μ_δ.	μ.
	′ ′							h m s	° ′	″	″	″
2	40×40	227	76	13·0	15·2	—						
1	50×50	296	141	13·1	14·9	—	11·4	7 3 32	+59 26·2	+0·091	−0·274	0·289
						240	6·7	7 7 11	+59 48·9	− ·116	− ·180	·214
						643	14·7	7 10 12	+60 15·9	+ ·136	− ·090	·163
5	60×60	174	80	12·9	14·8	555	11·3	15 17 29	+60 13·2	− ·158	− ·172	·234
						563	14·6	15 18 39	+60 15·2	− ·139	− ·342	·369
						161	11·8	15 20 28	+59 29·4	− ·039	− ·193	·197
7	40×40	301	101	12·7	14·8	—						

* See *M.N.*, **87**, 439.

Selected Area No.	Area measured.	No. of Stars measured.	No. of Comparison Stars.	Mean Mag. of Comparison Stars.	Limiting Magnitude.	Star No.	Magnitude.	R.A. 1900·0.	Dec. 1900·0.	Annual Proper Motion. μ_α.	μ_δ.	μ.
								h m s	° ′	″	″	″
33	60′×60′	157	58	13·1	15·2	138	14·2	13 47 33	+44 52·5	+ ·068	− ·090	·11
						267	9·4	13 50 7	+45 12·9	− ·134	+ ·022	·13
43	40×40	214	83	12·7	14·6	—						
58	60×60	155	52	13·1	15·5	—						
62	50×50	507	146	12·9	15·2	801	12·9	17 54 18	+30 8·0	+ ·127	·000	·12
						888	14·6	17 55 49	+30 1·4	− ·085	− ·140	·16
						221	7·6	17 56 48	+29 34·3	− ·149	+ ·139	·20
						226	13·4	17 56 55	+29 33·9	− ·145	+ ·180	·23
						1206	11·2	17 56 55	+30 19·2	− ·078	+ ·078	·11
66	50×50	358	174	12·8	14·5	348	12·3	21 57 42	+29 51·0	+ ·100	+ ·066	·12
96	50×50	236	110	12·9	14·7	871	13·5	4 46 51	+ 0 21·4	+ ·167	+ ·118	·20
101	60×60	202	94	13·1	14·8	216	12·2	9 53 10	− 0 15·0	+ ·052	− ·100	·11
107	60×60	178	66	12·7	14·7	684	8·6	15 32 10	+ 0 10·0	+ ·081	− ·063	·10
						580	12·2	15 33 22	+ 0 5·8	+ ·147	− ·201	·24
						595	7·7	15 33 55	− 0 1·0	− ·185	− ·110	·21
112	50×50	321	122	12·5	14·7	1805	11·4	20 37 59	+ 0 11·5	+ ·101	− ·046	·11
113	50×50	252	86	13·1	15·2	421	10·8	21 38 34	+ 0 5·5	+ ·073	− ·077	·10
115	60×60	153	66	12·4	14·6	331	10·9	23 37 0	+ 0 25·8	+ ·094	− ·104	·14
						496	12·6	23 37 17	+ 0 40·4	− ·052	− ·085	·10
						525	12·9	23 39 38	+ 0 40·9	+ ·101	− ·013	·10

Radcliffe Observatory, Oxford :
1929 *January* 1.

New Double Stars. By Rev. T. E. Espin and W. Milburn.

The arrangement of the lists of New Double Stars is similar to that of previous communications, and therefore needs no further explanation.

Espin. No.	B.D.	(1900). R.A.	(1900). Decl.	P.	D.	Mags.		Nts.	Date 1928.
	°	h m	° ′	°	″				
2329	+33·197	1 10·9	+33 16	127·7	3·85	9·5	13·0	2	·018
2330*		2 9·3	32 58	198·9	2·01	10·0	10·2	3	·047
2331	+36·659	3 7·3	36 26	124·7	1·94	9·5	10·6	4	·027
2332	+35·675	16·7	36 3	250·2	7·40	9·5	11·0	3	·067
2333	+36·748	39·5	36 16	150·8	5·65	9·5	11·7	2	·023
2334*		46·0	35 27	242·4	3·49	10·5	11·0	4	·098

Espin. No.	B.D.	1900. R.A.	1900. Decl.	P.	D.	Mags.	Nts.	Date 1928.	
	°	h m	° ′	°	″				
2335	+36·774	47·8	36 24	248·4	2·81	8·5 12·0	2	·023	
2336	+36·869	4 13·4	36 8	103·2	1·81	9·5 9·6	4	·075	
2337	+37·979	45·8	37 39	244·5	2·39	10·0 13·0	3	·034	BC
				116·5	24·55	A = 9·5	3	·049	AB
2338	+38·981	50·4	38 28	271·5	6·02	9·5 10·0	3	·047	
2339	..	59·5	37 20	340·5	3·51	10·7 10·7	2	·157	
2340	+37·1213	5 23·9	37 20	191·3	2·91	10·0 12·0	1	·027	BC
				356·8	40±	A = 9·3	1	·027	AB
2341	+37·1343	45·9	37 29	303·2	4·01	9·5 10·8	2	·026	
2342	+37·1442	6 6·4	37 43	241·2	7·16	9·3 13·0	2	·115	
2343	+37·1445	7·1	37 27	144·8	3·62	9·2 11·0	1	·159	
2344	+35·1681	7 40·9	35 42	42·7	3·70	9·5 10·5	2	·157	
2345	+34·1819	8 18·1	34 29	214·0	6·41	9·2 9·5	1	·238	
2346	+34·1866	30·9	34 41	190·6	2·21	9·5 10·7	3	·184	
2347	+32·1928	9 38·8	32 46	249·0	7·54	9·4 9·7	1	·238	
2348	..	48·3	32 45	312·5	2·16	10·0 11·0	1	·238	AB
				124·2	5·75	C = 12·5	1	·238	AC
2349	+33·3563	19 37·8	33 9	7·4	3·76	9·3 13·0	4	·833	
2350	+33·3723	20 1·0	33 17	211·6	2·78	9·2 11·5	3	·859	
2351	..	1·8	33 12	271·7	4·45	10·0 11·5	3	·876	
2352	+33·3741	2·9	33 11	230±	3±	12·0 12·0	1	·868	BC
				47±	30±	A = 8·0	1	·868	AB
2353	+32·3815	21·1	32 44	148·1	2·49	9·1 12·0	4	·882	
2354*	+33·2933	27·3	33 17	311·4	6·58	9·3 11·0	2	·696	
2355*	..	31·1	33 12	7·3	2·50	10·5 11·0	1	·717	
2356	+32·3978	49·1	32 38	240·2	5·53	9·0 13·0	3	·892	
2357	+32·4028	57·9	32 22	334·0	8·43	9·5 11·0	3	·889	
2358	..	57·9	32 25	193·4	5·22	11·0 11·1	2	·885	
2359	+31·4512	21 33·5	31 16	78·9	2·71	9·4 9·5	2	·829	
2360	..	51·5	31 13	262·1	3·76	10·5 10·5	2	·912	
2361	..	22 1·3	30 31	275·0	4·41	9·6 12·5	2	·932	
2362	+30·4861	56·7	30 31	171·2	2·00	9·7 10·2	3	·942	

Notes.

2330. Two stars here; the S. one is assumed to be B.D. +32°·406.
2334. Star, mag. 11·5, at P. 224°·3, D. 17″·9.
2354. M.P. 312°·5, D. 6·66, 9·3, 10·7, 1 nt. 1928·797.
2355. M.P. 9°·4, D. 2″·22, 1928·797.

Milburn. No.	B.D.	1900. R.A.	1900. Decl.	P.	D.	Mags.	Nts.	Date 1928.
	°	h m	° ′	°	″			
552	+29·8	0 4·7	+29 40	190·6	8·35	8·7 9·5	4	·953
553	+27·18	8·4	27 55	43·4	7·05	9·5 9·8	3	·884
554	+27·20	8·8	28 4	66·0	4·75	9·5 9·8	2	·788 AB
				48·3	24±	C = 11·5	1	·780 AC
555	..	50·1	29 18	107·9	4·96	9·8 11·5	2	·965
556	..	1 13·3	27 59	289·5	2·99	9·5 9·7	1	·808
557	+29·536	3 6·7	29 40	105·9	2·45	9·5 9·8	3	·071
558	..	4 6·0	27 49	355·8	8·20	9·5 9·7	1	·129
559	..	39·5	29 6	176·2	1·99	10·7 12·5	2	·042 AB
				346·9	40±	C = 10·5	2	·042 AC
560	..	55·7	28 51	68·0	4·78	9·6 12·1	3	·078
561	..	5 4·8	28 45	177·9	2·49	10·0 12·0	3	·039
562	..	34·0	28 24	189·5	3·03	11·0 12·7	2	·114
563	..	6 13·4	28 1	74·9	3·45	11·5 12·0	1	·071
564	+29·1277	28·8	29 7	237·4	4·17	9·5 10·5	2	·048 AB
				15·0	8·27	C = 10·6	2	·048 AC
				19·7	60±	D = 10·1	1	·071 AD
565	+28·1540	7 58·5	28 48	327·1	5·51	9·5 10·0	2	·157
566	+28·1546	59·4	28 47	163·5	2·03	9·5 12·0	2	·157
567	*h* 491	10 51·1	28 27	118·6	18·27	9·3 10·0	2	1927·318 AB
				250·3	4·01	C = 11·8	3	1927·313 BC
568	..	18 29·9	28 11	203·8	4·21	9·5 10·0	1	·539
569*	+29·3597	19 22·8	29 17	193·4	4·75	8·7 13·0	2	·672 AB
				215·7	6·00	9·5 11·5	3	·687 CD
				310±	25±	..	1	·643 AC
570	..	54·3	28 34	14·8	2·72	10·0 10·5	2	·709
571	..	20 27·5	29 9	264·9	2·95	9·7 11·2	2	·689
572	..	49·2	27 28	3·2	5·63	9·9 10·2	2	·823
573	..	53·7	29 17	236·7	4·98	10·2 10·3	2	·840 AB
				229·6	20±	C = 10·5	1	·851
574	..	21 12·4	27 54	334·3	8·08	9·5 10·5	2	·709
..	+27·4120	33·5	27 16	124·4	11·95	9·3 10·0	2	·892
575	..	37·8	29 41	96·0	2·49	11·3 12·5	2	·924 AB
				312·6	17±	C = 9·8	1	·931 AC
576	+27·4178	45·8	27 25	284·9	9·53	9·4 11·0	2	·773
577	+27·4201	51·1	27 53	25·3	5·67	10·0 11·5	2	·840 BC
				300·1	48±	A = 9·5	1	·830 AB
578	..	54·8	27 31	161·8	2·76	9·4 13·0	3	·875
579	+26·4349	22 2·4	27 6	27·1	8·50	9·5 9·7	3	·877
580	..	22·8	29 44	261·0	3·14	11·5 12·0	2	·947
581	+29·4675	24·7	29 37	240·6	2·76	9·5 11·5	2	·912 AB
				6·4	22±	C = 10·3	1	·917 AC

Milburn. No.	B.D.	1900. R.A.	1900. Decl.	P.	D.	Mags.	Nts.	Date 1928.
	°	h m	° ′	°	″			
582	+29·4685	0 27·2	29 22	340·6	4·67	9·5 10·0	2	·912
583	+28·4394	28·0	28 21	286·4	8·96	9·5 10·0	2	·749
584	..	32·1	27 1	306·6	4·94	10·0 10·2	2	·830 AB
				226·6	11±	C = 12·0	1	·808 BC
585	..	37·7	26 32	94·5	9·27	9·5 10·5	2	·790
586	..	46·3	29 30	9·4	6·84	10·5 11·5	2	·932 AB
				352·3	45±	C = 10·5	1	·931 AC
587	..	23 8·8	29 26	256·0	3·98	10·0 10·5	2	·947
588	..	24·2	29 28	197·4	9·19	9·6 9·9	2	·923
589	+26·4659	30·9	26 42	226·4	8·12	9·5 9·8	2	·879
590	..	42·4	29 24	169·5	2·60	10·0 12·7	3	·948
591	..	58·0	30 0	345·2	4·06	9·8 10·2	2	·943

Note.

569. I believe there is a closer and fainter *comes* of 14 mag. at 0°±, 4″, but I was unable to measure it.

Stars Missing or Faint.

B.D.	h m s	° ′			
°+31·246	1 17 20·4	+31 49·7	9·5	1928 Jan. 18.	Missing. E.
+35·652	3 5 59·3	35 59·7	9·5	Jan. 10.	Not seen. E.
+29·681	4 6 57·0	29 0·4	9·3	Jan. 10, 11, 14.	Missing. M.
+29·684	4 7 15·1	29 0·4	9·5	Jan. 10.	Two faint stars a little S. of place. M.
+36·922	4 29 52·5	36 34·5	9·5	Jan. 11.	Not seen. E.
+37·1088	5 3 16·3	37 44·7	9·5	Jan. 27.	Not seen. E.
+37·1246	5 26 8·4	37 16·3	9·5	Jan. 27.	Not seen. E.
+37·1623	6 48 12·8	37 42·7	9·4	Feb. 27.	Not seen, some haze. E.
+28·1407	7 21 50·8	28 31·6	9·5	Feb. 27.	Missing. Marked ? in Catalogue. M.
+34·1812	8 13 55·5	34 49·0	9·5	Mar. 28.	Not seen. E.
+28·1652	8 40 56·3	28 53·4	9·5	Feb. 28, Mar. 28.	Missing. M.
+34·1905	8 42 21·2	34 40·9	9·3	Feb. 27.	Not seen. A star at +34° 45′ mag. 9·3. E.
+28·3554	19 52 45·9	28 27·6	9·5	Aug. 23.	Wide faint pair at 26′·7. Noted as missing 1927. M.
+32·4096	21 8 15·4	32 18·8	9·3	Nov. 13.	Decl. +32° 21′. E.
+30·4473	21 28 29·8	30 7·8	9·5	Dec. 7.	13 mag. E.
+29·4497	21 38 20·9	29 49·3	9·5	Nov. 26.	Only faint stars here. M.
+30·4821	22 44 50·2	30 3·5	9·5	Dec. 1.	No star. One 3′ S. E.
+30·4949	23 20 52·6	30 14·4	9·5	Dec. 6.	No star here above 12·5. E.

Note on the Deviation of Stellar Atmospheres from Thermodynamic Equilibrium. By B. P. Gerasimovič.

It is well known that existing ionization theory fails to explain satisfactorily some of the data of stellar spectroscopy. The abnormal behaviour of hydrogen, the spectra of anomalous late type stars, and the high excitation spectra of early type stars with abnormally low effective temperatures are examples of our ignorance of the physical conditions of stellar atmospheres. Another more general and very important fact was brought to light by the recent work of Adams and Russell ; * there appears to be a conspicuous and widespread deviation from thermodynamic equilibrium, which makes the relative number of atoms in higher quantum states much larger than it should be from simple statistical theory. The logarithm of the relative number of atoms in different quantum states is not a linear function of the corresponding energy of excitation ; the relation exhibits a curvature which cannot be referred to inherent inaccuracy of data. It is remarkable that this anomaly does not affect the ionization, but only the distribution of the atoms between the various energy levels, signifying the failure of the Boltzmann law under the conditions prevailing in stellar atmospheres.

The usual formulæ characterizing the statistical equilibrium of a microscopic assembly (the Saha formula, and Boltzmann's law) certainly cannot be applied without additional analysis to stellar atmospheres, because the upper photosphere and the adjacent reversing layer are not in thermodynamic equilibrium. Obvious sources of a departure from the Boltzmann distribution are the normal temperature gradient that exists in stellar atmospheres—the difference in temperature between electrons and the stimulating photospheric radiation. Besides these " normal " deviations from the Boltzmann law, there is a possibility of much more important deviations caused by an abnormal energy distribution in the spectrum of the exciting radiation. Some very striking early type stars can be quoted † which show a large discordance between the temperatures derived by spectrophotometry and from ionization theory. A considerable abnormal increase in photospheric radiation in the far ultra-violet is indicated. In late type stars the abnormality might be caused by the direct obstructing action of bands, lines, or continuous absorption in the lower layers of the atmosphere, or by their indirect action in disturbing radiative equilibrium and distorting black body distribution. It therefore seems to be of importance to investigate the cause of the abnormality found by Adams and Russell, and to evaluate the contribution of band and line absorption in quantitative form.

The first step is the derivation of a generalized Boltzmann formula

* Adams and Russell, *Ap. J.*, **68**, 9, 1928.

† Kienle *M.N.*, **88**, 700; Greaves, Davidson, and Martin, *M.N.*, **88**, 703, 1928; Gerasimovič, *H.C.B.*, No. 857, 1928.

for an atmosphere with a temperature gradient. This is done in section 1, on certain reasonable assumptions. It is further shown that a temperature gradient, however large, cannot produce noticeable deviations from the Boltzmann law. The investigation is then turned in another direction, and it is shown that for red supergiants, for which the numerical data are sufficiently reliable, the abnormal increase of the number of atoms in the higher quantum states may be attributed to abnormalities in the exciting radiation caused by direct or indirect effects of absorption lines and bands.

1. *Distribution of Atoms among Quantum States in an Atmosphere with a Temperature Gradient.*

As soon as we reject the extremely useful Principle of Detailed Balancing, which enables us to investigate in a simple and elegant manner the condition of assemblies in thermodynamic equilibrium, we face an extremely complicated problem, which can be approximately solved only because the actual deviations of stellar atmospheres from thermodynamic equilibrium are not large. We therefore undertake the investigation of the problem, under the following explicit assumptions, which appear to be fairly reasonable :—

(i) The distribution of the velocities of the electrons in the reversing layer is spherical and Maxwellian ; in the upper chromosphere, because of increased radiation-pressure, the distribution may be ellipsoidal.

(ii) In the most general case of statistical equilibrium the total number of transitions of all kinds into a given state is balanced by the total number of inverse processes. If a_{ki} denotes the transition coefficient for an electron jump from the k orbit to the i orbit, and n_i refers to the number of atoms in the i state, we have as the condition for statistical equilibrium for this state

$$n_i(a_{i1} + a_{i2} + \ldots) + \text{number of ionizations from } i \text{ orbit} = n_1a_{1i} + n_2a_{2i} + n_3a_{3i} + \ldots + \text{number of captures into } i \text{ orbit.}$$

The coefficients a_{ki} are linear functions of the atomic quantities, and of the intensity of the stimulating radiation of the appropriate frequency with the appropriate dilution factors. If the deviations from thermodynamic equilibrium, and therefore from detailed balancing, are not large, this equation of statistical equilibrium can be simplified. In this case we may assume that the total number of ionizations from the given orbit is equal to the number of corresponding captures. Therefore, for a given state, the systems of interorbital jumps and the ionizations and recaptures are separately balanced when the material is in a steady state. The jumps of bound electrons from one orbit to another are, so to speak, events in the domestic life of the atom; ionizations and recaptures are, on the other hand, its only means of communicating directly with the external world of free electrons, at least at the temperatures and densities that prevail in stellar atmospheres. We therefore assume that in its external behaviour an atom

is not influenced by its bound electrons—an approximation that seems to be better than the Principle of Detailed Balancing. For every quantum state we now have two equations :

$$n_i(a_{i1} + a_{i2} + a_{i3} + \ldots) = n_1 a_{1i} + n_2 a_{2i} + n_3 a_{3i} + \quad . \qquad \text{(A)}$$

$$\text{number of ionizations} = \text{number of captures} \; . \qquad \text{(B)}$$

The second equation gives n_i independently of the unknown quantities a_{ki}.

(iii) As a first approximation we neglect the ionizing power of the radiation of the reversing layer " from above," in comparison with that of the mighty stream of radiation " from below." We neglect also stimulated captures and ionization by collision. We assume that the stimulating radiation is black body radiation diluted by a factor W, which is constant for the whole spectrum, and nearly equal to $\frac{1}{2}$. The data of observation show that this assumption is not legitimate for the conditions in stellar atmospheres ; we shall correct for it empirically.

We now put our condition into mathematical form ; for every i quantum state the total number of radiative ionizations is equal to the total number of captures. If n_i is the required number of atoms in the i state, $\mathrm{WI}(\nu)$ the intensity of the exciting radiation of temperature T, $\phi_i(\nu) \times \mathrm{I}(\nu)$ the probability of ionization by ν radiation from the i state, $\mathrm{F}_i(v)$ the corresponding function for the capture of an electron with velocity v, the number of ionizations from the i state for frequencies between ν and $\nu + d\nu$ will be

$$n_i(1 - x)\phi_i(\nu)\mathrm{WI}(\nu)d\nu,$$

and the corresponding number of captures for velocities between v and $v + dv$ will be

$$8\pi^2 x^2 \mathrm{N}\left(\frac{m}{2k\mathrm{T}_0\pi}\right)^{\frac{3}{2}} e^{-\frac{mv^2}{2k\mathrm{T}_0}} v^3 \mathrm{F}i(v)dv,$$

where T_0 is the temperature of the layer, and $\mathrm{N}x$ is the number of ionized atoms. The usual notation for physical constants has been used.

Condition (B), relating number of ionizations to number of captures, becomes

$$n_i(1 - x)\int_{\nu_i}^{\infty} \phi_i(\nu)\mathrm{WI}(\nu)d\nu = 8\pi^2 x^2 \mathrm{N}\left(\frac{m}{2\pi k\mathrm{T}_0}\right)^{\frac{3}{2}} \int_0^{\infty} e^{-\frac{mv^2}{2k\mathrm{T}_0}} v^3 \mathrm{F}_i(v)dv \qquad (1)$$

For the statistical functions we adopt the following expressions of Milne * and Kramers † :—

$$\phi_i(\nu) = \frac{4\pi^2 m^2 c^2 v^2 \mathrm{F}_i(v) q_1' \sigma}{h^3 \nu^3 q_i} ; \qquad \mathrm{F}_i(v) = \frac{\text{const.}}{v^2 \nu},$$

where the constant is independent of v or ν. Taking for $\mathrm{I}(\nu)$ Planck's

* Milne, *Phil. Mag.*, **47**, 209, 1924.
† Kramers, *ibid.*, **46**, 890, 1923.

function for the temperature T of the photospheric radiation, and performing the necessary integrations, we obtain

$$\frac{n_i(1-x)m^2\sigma q_1' W}{h^2 g_i}\sum_{j=1}^{j=\infty} E\left(-\frac{h\nu_i j}{kT}\right) = \frac{Nx^2 m^{\frac{1}{2}}h}{(2\pi kT_0)^{\frac{3}{2}}} e^{\frac{h\nu_i}{kT_0}} E\left(-\frac{h\nu_i}{kT_0}\right).$$

In this expression $E(-z) = \int_{\infty}^{1} \frac{e^{-zt}dt}{t}$, and is an integral logarithm. This gives

$$\frac{n_i}{n_1} = \frac{q_i}{q_1} e^{\frac{h(\nu_i - \nu_1)}{kT_0}} E\left(-\frac{h\nu_i}{kT_0}\right)\sum_{j=1}^{j=\infty} E\left(-\frac{h\nu_1 j}{kT}\right) \Big/ E\left(-\frac{h\nu_1}{kT_0}\right)\sum_{j=1}^{j=\infty} E\left(-\frac{h\nu_i \cdot j}{kT}\right)$$

For sufficiently large values of the argument we have the expansion

$$E(-z) = -\frac{e^{-z}}{z}\left[1 - \frac{1}{z} + \frac{2!}{z^2} - \ldots\right] \quad . \quad . \quad (2)$$

Let the relative number of atoms in the i and 1 quantum states, as determined by the generalized theory, be

$$\frac{n_i}{n_1} = \left(\frac{n_i}{n_1}\right)_0 . f_i,$$

where $(n_i/n_1)_0$ is the corresponding number calculated from Boltzmann's law at the temperature T of the stimulating radiation, and f_i is the correcting factor, which is a function of T and also of T_0, the temperature of the atoms considered. The correcting factor tends to unity as T_0 tends to T. Now, using expansion (2) and formula (1), and remembering that $h\nu_i$ is the energy of the i level, and $h\nu_1$ the ionization potential, we have

$$f_i = \frac{\left(1 - \frac{kT}{h\nu_1} + 2\left(\frac{kT}{h\nu_1}\right)^2 - \ldots\right)\left(1 - \frac{kT_0}{h\nu_i} + 2\left(\frac{kT_0}{h\nu_i}\right)^2 - \ldots\right)}{\left(1 - \frac{kT}{h\nu_i} + 2\left(\frac{kT}{h\nu_i}\right)^2 - \ldots\right)\left(1 - \frac{kT_0}{h\nu_1} + 2\left(\frac{kT_0}{h\nu_1}\right)^2 - \ldots\right)}$$

$$= 1 + \frac{k(T_0 - T)}{h}\left(\frac{1}{\nu_1} - \frac{1}{\nu_i}\right) + \ldots$$

provided that the terms of the form $h\nu/kT$ are so large that we can neglect second and higher powers of $e^{-\frac{h\nu_i}{kT}}$, etc. Moreover, in most cases we can also neglect the squares of $kT/h\nu$, and give our formula the final form shown above.

One important remark should be made. Eliminating n_i from (A) and (B) we obtain the equations from which to determine the quantities a_{ki} as functions of the temperatures T, and T_0, and the dilution factors, for all lines in the spectrum. Suppose that owing to some perturbation the radiation of the photosphere or the dilution factor for some special line changes: then, in order to keep constant the atomic quantities (which enter into a_{ki}), the temperature T_0 and the dilution factors of all

other lines must change—as soon as the perturbation happens the gas begins to move until it reaches a part of the reversing layer where T_0 and the dilution or absorption of exciting radiation for the separate lines and the continuous spectrum have special values capable of maintaining the steady state defined by (A) and (B).

It may be objected that, in determining the value of n_i/n_1 from ionizations and captures only, we make it independent of the absorption of the corresponding principal line, which lifts electrons up to the i level. As we have said, the absorption of this line has already played its part in pushing the atoms to the special pressure level where the temperature is T_0; in a steady state therefore we need not take this line into account a second time. If we knew all the a_{ki} values, the equation (A) would give us the same values of n_i/n_1 as equation (B), which we used above.

Obviously the deviations from Boltzmann's law increase as the temperature difference increases, and are smaller for late type stars. On the other hand they are in the direction observed—the number of atoms in the higher states is larger than in thermodynamic equilibrium. It is easy to interpret this result. The number of ionizations per atom will be less than the number in thermodynamic equilibrium by a factor that is constant for all levels—the dilution factor of the radiation, which is very near to $\frac{1}{2}$. But the number of captures in a given state increases with decreasing temperature owing to the diminished speed of the electrons, this increase being smaller for lower orbits. In order to preserve the statistical equilibrium for all levels, the number of atoms in lower states should be less numerous than that in higher states.

2. *The Anomaly in Late Type Stars.*

We now calculate the value of n_i/n_1 for the various excitation levels of iron, adopting, according to Russell,* $hv_1 = 7{\cdot}83$ volts, and taking the following sequence of excitation potentials hv_i:—

i.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
hv	7·77	6·89	6·29	5·66	5·27	4·94	4·59	4·36	3·80	3·33

Consider as an illustrative example an M supergiant like α Orionis, taking $T = 3000°$, and $T - T_0 = 0{\cdot}12\,T$, in accordance with the theory of radiative equilibrium. Calculation shows that the values of $f_i - 1$ are quite negligible. For $i = 10$, $f_1 = 1{\cdot}0054$, and even for effective temperature 30,000°, $f_i = 1{\cdot}054$. We see, therefore, that a temperature gradient in the stellar atmosphere, however reasonably large, cannot suffice to produce noticeable deviations from the Boltzmann distribution law. To explain the anomaly found by Adams and Russell we must make a search in another direction.

According to Adams and Russell, ionization is unaffected by the anomaly under discussion. On the other hand, their diagram for the iron lines in α Orionis and α Scorpii, as compared with the sun, seems to show that the observations for states one to five, inclusive, can be

* Russell, *Ap. J.*, **66**, 233, 1927.

represented approximately by a straight line with a slope $-0{\cdot}84$. If $T_\odot$ is the effective temperature of the sun, and T that of the star compared with it, the slope of the line is equal to $5040\left(\frac{1}{T_\odot}-\frac{1}{T}\right)$, representing the logarithm of the relative number of atoms in the various states as a function of excitation potential. If we take $T_\odot$ as 6000, we obtain $T = 3000$, quite a reasonable effective temperature for a red supergiant. Thus the linear solution for the first to the fifth state gives no indication of the anomaly, which begins to play a marked rôle between the fifth and sixth states, increasing for higher states, so that for the tenth orbit the relative number of atoms is about ten times the value that would be given by a straight line. The anomaly of the intensity curve of the exciting radiation therefore begins somewhere between 2300 and 2500 A, and extends towards the red.

Let us consider this case further. Suppose that there exists between ν_9 and ν_{10} an absorption band of infinite "depth," the corresponding block of radiation will be cut out of the spectrum altogether, thus producing a change in the correcting factor f_{10}. To take this into account it is sufficient to change slightly the equation of statistical equilibrium for $i = 10$, the upper limit of the integral on the left side of equation (1) becoming ν_9 instead of ν_{10}. In this case we get

$$f_{10} = e^{\frac{h(\nu_9-\nu_{10})}{kT}}\frac{\nu_{10}}{\nu_9}\left(1+\frac{kT}{h}\left(\frac{1}{\nu_9}-\frac{1}{\nu_{10}}\right)\right).$$

The hypothetical band will therefore produce a sudden rise in the relative number of atoms at the tenth state, and in the atmosphere of α Orionis they will be seven times as numerous as they would have been in the absence of the band. This is evidently due to a decrease of ionizations from this state owing to a lack of the most effective ionizing radiation between ν_9 and ν_{10}.

This idealized example encourages us to carry the analysis further. Suppose that the stellar radiation reaching the level of the iron atoms has been reduced by selective absorption, or otherwise, by the factor $\alpha_{5,6}<1$ between ν_5 and ν_6, by the factor $\alpha_{6,7}$ between ν_5 and ν_6, and so on. To calculate the relative number of atoms in the sixth state we must use the dilution factor W for radiation between $\nu = \infty$ and $\nu = \nu_5$, and $W\alpha_{5,6}$ for the radiation between ν_5 and ν_6. If we neglect the factors of the order of $\left(1-\frac{kT}{h\nu}\right)$, we obtain

$$f_6 = \left\{\left[(1-\alpha_{5,6})\frac{\nu_5}{\nu_6}e^{\frac{h(\nu_6-\nu_5)}{kT}}\right]+\alpha_{5,6}\right\}^{-1}.$$

We can find an analogous but more complicated expression for f_7 involving $\alpha_{5,6}$ and $\alpha_{6,7}$, and so forth. Since the data are relative to the sun, any deviation of solar radiation from that of a black body would affect the results. We shall assume that the radiation of the sun does not differ sensibly from black body distribution. Then we can derive appropriate values of α from the data of Adams and Russell,

and thus construct an approximate intensity curve for short wave-lengths, otherwise inaccessible to us, which would explain the observed deviation from normal distribution among the states of the atom on the basis of the preceding theory. Neglecting a small " normal " deviation from the Boltzmann distribution due to the temperature gradient, we find the following distribution for a red supergiant, based on the observed data for the mean of α Orionis and α Scorpii given in the paper by Adams and Russell :

i.	$\Delta(\log f_i)$.	$a_{i-1,\,i}$.
6	+0·21	0·44
7	0·43	0·29
8	0·44	0·35
9	0·78	0·14
10	+0·93	0·11

The second column, headed $\Delta(\log f_i)$, contains the increase in the logarithm of f_i, compared with the straight line that it would follow in thermodynamic equilibrium at T = 3000°.

We have not taken into account the difference in pressure level for lines of different energy levels, which tends to make the curve vary a little more steeply with wave-length. We can now trace the deviations from a black body curve that are required to account for the anomaly in the distribution of the iron atoms. The numerical results, expressed in stellar magnitudes, are given below :

Wave-length.	Magnitudes. m
2380 A	+0·9
2500	1·3
2750	1·1
3030	2·1
3470	+2·4

It is well known that the intensity of the spectra of M stars in the visual region is considerably reduced by band and line absorption. According to Joy * the absorption by titanium-oxide bands in o Ceti is represented by a factor 6, corresponding to a reduction in light of $1^m{\cdot}9$, in the region 4300–6700, compared to that for the region from 4100 to 4300. Although this result has been shown to be exaggerated,† a strong absorption in the far ultra-violet region of red supergiants would not be very surprising. It would of course be too hypothetical and futile to try to identify the bands at present ; but the investigation of the spectra of red supergiants in the far ultra-violet is a very important problem. The anomaly affecting the tenth state of the iron atom is to be explained, on our assumptions, by heavy bands or accumulations of lines between 3030 and 3700, and unless such bands are concentrated in the short-wave side of this interval of spectrum, they are

* Joy, *Mount Wilson Contr.*, No. 311, 1926.
† Payne and Hogg, *H.C.C.*, 308, 1927.

certainly not impossible to detect. The existing data from stars of Class K to Class A are too scanty to be of value in this matter. The Adams-Russell results suggest that the anomaly is present for Arcturus, but not noticeably for the earlier stars. This seems to corroborate our assumption that the anomaly for the sun is small, and can be neglected in comparison with that for a red supergiant.

One important point must be considered further. Atoms can be put into the i state in two different ways : by captures, form (B), or by interorbital jumps, form (A). According to our hypothesis these two mechanisms work independently and give the same result. Interorbital transitions involve only relative values of n_i. How can the increase in relative n_i be produced by the interorbital transitions ? This question cannot be answered quantitatively, because the statistical equilibrium for such transitions depends on several unknown quantities : the probability coefficients and dilution factors for the various lines. In the simplest case, where only two states are present, the answer is evident. The lifting of the electron to the second state is accomplished by the absorption of a single line ; here the abnormal abundance of the electrons in the second state is to be explained by the abnormal intensity of the exciting radiation. But in the actual case the conditions are much more complicated ; the lifting of electrons into higher states by absorption of principal lines is only one factor, though probably the most important. To explain the abnormal relative n_i/n_1 for states 6 to 10, we must suppose that the radiation corresponding to the first to fifth principal lines is less intense (referring to 3000° as the standard temperature) than that corresponding to the sixth to tenth principal lines. It is evident that in this case the *relative* number of atoms in higher states will be abnormally great. The first five lines are located between wave-lengths 2 μ and 0·48 μ, and the last five between 0·43 μ and 0·27 μ. Everything will be explained if the infra-red, red, and green parts of the spectrum are more affected by bands than the violet and ultra-violet. As a matter of fact, the violet and near ultra-violet regions of red stars are relatively clear of bands, but there are two principal lines (9 and 10) that are excited by radiation which would be affected by the hypothetical bands suspected in the far ultra-violet. The contradiction could be removed if there were some clear space between the heavy bands, corresponding to the radiation needed to excite the lines in question.

It should be clearly stated that the absorption calculated above refers to radiation exciting the iron atoms at the levels at which the iron absorption lines are formed. If our explanation of the deviations from thermodynamical equilibrium in the atmospheres of red supergiants is correct, it seems to suggest that the bands are formed at levels of higher pressure than are the iron lines. Unfortunately our data on the relative pressure levels are still scanty, even contradictory. Iron seems to be a " low-level " element. On the other hand, the enormous abundance of iron in stellar atmospheres * tends to make the effective pressure for the iron arc lines lower. Some observations show that the

* Payne, *Stellar Atmospheres*, 188, 1925.

electron pressure as deduced from the iron lines in some stars is only a little higher than that deduced for ionized strontium.*

Besides the direct effects of absorption, the bands will have a further indirect effect on the continuous spectrum by redistributing the energy absorbed, and thus altering the distribution of intensity in the undisturbed continuous spectrum.† As we know nothing of the bands in the infra-red and far infra-red, we cannot calculate this effect, but very possibly it acts in the same direction as selective absorption in the ultra-violet.

The well-known anomaly of the hydrogen atoms remains unexplained; the possible values for higher frequencies are too small to supply a millionfold more atoms in the Balmer state than the theory predicts. It is hardly necessary to say that the anomalies in early stars can be explained by abnormal behaviour of the radiation of the highest frequencies, quite different from that investigated in this paper.

Occultations of Stars by the Moon, observed at the Astronomical Observatory, Kiew, in the Year 1928. By Professor S. D. Tscherny.

(*Communicated by the Secretaries.*)

No.	Date 1928.	G.M.T. h	m	s	Star.	Mag.	Dis. or Reap.	Telescope and Power.	Notes.
1	Feb. 27	18	47	44·54	43 Tauri	5·5	D	R145	Instantaneous.
2	27	19	50	24·99	43 Tauri	5·5	R	R145	Satisfactory.
3	Mar. 27	20	19	59·49	412 B Tauri	5·8	D	R145	Good.
4	Apr. 28	18	26	13·94	42 Leonis	6·1	D	R145	Good.
5	May 2	17	56	28·78	65 Virginis	6·0	D	R145	Satisfactory.
6	2	18	53	30·71	66 Virginis	5·7	D	R145	Good.
7	2	19	2	37·21	65 Virginis	6·0	R	R145	Doubtful.
8	22	18	43	10·42	52 Geminorum	6·1	D	R145	Good.
9	Aug. 29	20	36	25·44	143 B Capricorni	6·1	D	R145	Instantaneous.
10	Sept. 29	17	42	31·31	37 Geminorum	5·7	D	R145	Satisfactory.
11	29	18	7	47·16	37 Geminorum	5·7	R	R145	Good.
12	29	19	12	50·97	40 Geminorum	6·3	D	R145	Doubtful.
13	29	19	51	8·19	40 Geminorum	6·3	R	R145	Satisfactory.

Instrument used :

R = 243 mm. Merz-Repsold refractor.

Astronomical Observatory, Kiew :
1929 *January* 2.

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* Gerasimovič, *H.C.C.*, No. 311, 1927.
† Milne, *M.N.R.A.S.*, **81**, 510, 1921.